
Contents

Preface	ix
 Chapter 1. Holomorphic Functions of Several Complex Variables	1
1.1. Notations, definitions and properties	1
1.2. Cauchy integral formula and applications	7
1.3. Power series in several variables	9
1.4. Various fundamental properties	20
1.5. Inverse mapping and implicit function theorems	28
1.6. Exercises	32
 Chapter 2. Complex Manifolds and Analytic Varieties	37
2.1. Preliminaries	37
2.2. Examples of complex manifolds	42
2.2.1. $\mathbb{C}^n$ and open subsets	42
2.2.2. Complex algebraic curves or compact Riemann surfaces	43
2.2.3. Spheres	43
2.2.4. Projective spaces	47
2.2.5. Grassmannians	50
2.2.6. Tori	51
2.3. Tangent spaces and tangent bundles	52
2.4. Constant rank theorem	53
2.5. Submanifolds, subvarieties and examples	55
2.6. Exercises	58

Chapter 3. Sheaves and Cohomology	65
3.1. Sheaves	65
3.2. Čech cohomology	69
3.3. De Rham cohomology	71
3.4. Dolbeault cohomology	72
3.5. Connections and curvature	74
3.6. Curvature form, first Chern class of line bundles	76
3.7. The Poincaré dual	77
3.8. Exercises	78
Chapter 4. Divisors and Line Bundles	79
4.1. Divisors	79
4.2. Line bundles	81
4.3. Sections of line bundles	83
4.4. Tori and Riemann form	89
4.5. Line bundles on complex tori	91
4.6. Exercises	97
Chapter 5. Some Fundamental Theorems	101
5.1. Preliminaries and various notions	101
5.1.1. Projective variety	101
5.1.2. Tangent and cotangent bundles	103
5.1.3. Dolbeault cohomology group	103
5.1.4. First Chern class	106
5.1.5. Hodge forms, hermitian metrics, harmonic space, Hodge star operator and Hodge theorem	107
5.1.6. Kähler metric, Kähler form, Kähler manifold	109
5.2. The Kodaira–Nakano vanishing theorem	111
5.3. The Lefschetz theorem on hyperplane sections	113
5.4. The Lefschetz theorem on $(1, 1)$ -classes	114
5.5. The Kodaira embedding theorem	117
5.6. Exercises	126
Chapter 6. Abelian Varieties	133
6.1. Definitions and examples	133
6.2. Riemann conditions, polarization and Riemann form	138
6.3. Isogenies and reducible Abelian varieties	141
6.4. Dual Abelian varieties	142
6.5. Prym varieties	145
6.6. Projectively normal embedding	154
6.7. Number of even and odd sections of a line bundle	155
6.8. Exercises	156

Chapter 7. Theta Functions and Complex Projective Tori	159
7.1. Meromorphic functions and theta functions	159
7.2. Lefschetz theorem	173
7.3. Exercises	179
 Chapter 8. Algebraically Completely Integrable Systems	183
8.1. Preliminaries	183
8.2. The Hénon–Heiles system	190
8.3. Kowalewski’s spinning top	204
8.4. Kirchhoff’s equations of motion of a solid in an ideal fluid	213
8.5. Exercises	216
 Chapter 9. Appendix: Riemann Surfaces and Algebraic Curves	227
 Chapter 10. Appendix: Elliptic Functions and Elliptic Integrals	233
10.1. Elliptic functions	233
10.2. Weierstrass functions	235
10.3. Elliptic integrals and Jacobi elliptic functions	240
10.4. Application: simple pendulum	244
 References	247
 Index	251