
Contents

Introduction	xiii
Chapter 1. A Global Method for the Scattering by a Multimode Plane with Arbitrary Primary Sources and Complete Series with Error Functions	1
1.1. Introduction	1
1.2. Potentials, reflection coefficients and multimode boundary conditions	2
1.2.1. Fields and potentials	2
1.2.2. Multimode boundary conditions for a multilayer backed by an impedance plane	4
1.2.3. Extended multimode boundary conditions	5
1.2.4. Properties of $R_{e,e}$ and $R_{h,h}$, and consequences on the $g_j^{e(h)}$	7
1.3. Incident potentials $(\mathcal{E}_i, \mathcal{H}_i)$ for arbitrary bounded primary sources	8
1.4. Scattered potentials $(\mathcal{E}_s, \mathcal{H}_s)$ for arbitrary primary sources	9
1.4.1. A global expression of $(\mathcal{E}_s, \mathcal{H}_s)$ for a multimode plane	9
1.4.2. Reciprocity principle from our expressions	11
1.5. When primary sources J and M are arbitrarily oriented dipoles	12
1.5.1. Tensorial expressions of fields	12
1.5.2. Exact reduction of derivative orders after sums over ϵ' indices	15
1.6. Integral reduction of \mathcal{J}_g and exact remarkable expansion with error functions	18
1.6.1. A non-singular integral for \mathcal{J}_g with $g = \sin \theta_1$ as $ \text{Re}(\theta_1) \leq \pi/2$	18
1.6.2. Exact remarkable expansion of $\mathcal{M}(a, b)$ with error functions	19
1.6.3. Exact expansion of $\mathcal{J}_g(\rho, -z)$ for arbitrary parameters	22

1.7. Asymptotic expansion of \mathcal{J}_g with complementary error functions	23
1.7.1. Asymptotics of $\mathcal{M}(a, b)$	23
1.7.2. Asymptotics of \mathcal{J}_g for large $ k_0 \rho \cos \theta_1 $ or $ k_0 R(1 + \sin(\theta_1 \mp \varphi)) $	27
1.8. Forward and backward guided waves	29
1.8.1. Guided waves from asymptotics (1.92) and (1.94)	29
1.8.2. Guided-like waves from the exact expansion (1.75)–(1.76)	30
1.9. References	30
1.A. Appendix A: complex spectral properties	33
1.A.1. Wave spectrum and reflection coefficients	33
1.A.2. Complex Poynting vector and reflection coefficients	34
1.B. Appendix B: some equalities derived from (1.4) and (1.11)	35
1.C. Appendix C: other types of bounded sources	35
1.C.1. Bounded volume sources	35
1.C.2. Bounded surface sources	36
1.D. Appendix D: some miscellaneous results on $\mathcal{J}_g(\rho, -z)$ and $W_m(s)$	36
Chapter 2. Diffraction by an Impedance Curved Wedge with Arbitrary Angle and Uniform Higher Order Asymptotics	39
2.1. Introduction	39
2.2. Higher order asymptotics in the region without creeping waves terms	40
2.2.1. Spectral function and asymptotic boundary conditions on both faces	40
2.2.2. Detailed calculation of first term influenced by the curvatures	44
2.3. Several orders asymptotic expressions in a region with creeping waves terms	48
2.3.1. Plane wave illumination: observation at infinity	48
2.3.2. Plane wave illumination: observation at finite distance	49
2.3.3. Source and observation points at finite distance	51
2.4. Some developed expressions concerning $f = \sum_{0 \leq n \leq m} f_n/k^n$ for arbitrary wedge angle	53
2.4.1. A developed expression of χ for $f_1(\alpha) = \Psi(\alpha)\chi(\alpha)$	53
2.4.2. Perfectly conducting case: curved half plane and discontinuity of curvature	56
2.4.3. Discontinuity of curvature in an impedance surface	57
2.5. Rewriting the field with fringe waves of second order	58
2.5.1. Definition of fringe waves u_{fr} when $ \varphi < \Phi$	58
2.5.2. The field term u_{opa}^\pm and the expression of u_{fr} at large distance	60
2.5.3. The expressions of u_{fr} for $ \varphi_o < \Phi$ and $ \varphi < \Phi$	61
2.5.4. The term of radiation u_{op}^\pm derived from u_{OGe}	62

2.5.5. Continuation of $u_{fr}^\pm(\rho', \varphi', \rho, \varphi)$ in shadow zone when $\pm\varphi' > \Phi$	65
2.6. References	66
2.A. Appendix A: on an integral expression with asymptotic kernel for the field near tangency of faces at $\varphi = \pm\Phi$, permitting an explicit transition at arbitrary order	67
2.A.1. Expression when the far field function F has no singularity	67
2.A.2. Expression when F has a singularity	72
2.A.3. Why we need to consider $\sin \theta = \sin \theta^\pm$ for the transition function at $\varphi = \pm\Phi$	73
2.B. Appendix B: first-order asymptotic of transition integral above tangent faces $\varphi = \pm\Phi$	74
2.B.1. When F has no singularity	74
2.B.2. When F has a simple real pole	75
2.C. Appendix C: regularity of a sum of residues at coalescent poles	77
2.D. Appendix D: analytical expression of u_{OGa} from behavior at reflection poles of f	78
Chapter 3. Spectral Equations for Scattering by Impedance Polygons: Properties and Solutions	81
3.1. Introduction	81
3.2. Generalities	81
3.3. Single-face expression of f for a scatterer enclosed by a surface with two semi-infinite polygonal faces	84
3.4. Polygonal surface with impedance boundary conditions	86
3.4.1. Functional equations on f_{e,m^\pm}^\pm due to conditions on semi-infinite planes	86
3.4.2. Functional equations due to boundary conditions on finite segments	87
3.4.3. A remarkable relation between $f_{a,p}^\pm$ and $f_{b,p}^\pm$ deriving from (3.17) when (3.10) applies	88
3.5. Formulation of the three-part polygonal problem: spectral functions in Sommerfeld–Maliuzhinets representation and functional difference equations in complex plane	89
3.5.1. Definition	89
3.5.2. About functional difference equations in complex plane	90
3.5.3. Functional difference equations for $f_{br}(\alpha) = f_b(\alpha + \frac{\Phi_b}{2})$, $f_{ar}(\alpha) = f_a(\alpha - \frac{\Phi_a}{2})$	92
3.6. The integral expressions and integral equations for the three-part impedance polygon	93
3.6.1. Elementary integral solutions for difference equations	93
3.6.2. Coupled integral expressions for $f_{br}(\alpha)$ and $f_{ar}(\alpha)$	94
3.6.3. Integral equations when $\Phi_{a,b} > -\frac{\pi}{2}$, as $ \arg(ik) < \pi/2$	95

3.6.4. Modification of integration path, and extension including k real	97
3.7. Existence and uniqueness for the integral equations on \mathcal{C}_ϵ	102
3.7.1. Uniqueness for the integral equations as $ \arg(ik\Delta) \leq \pi/2$	102
3.7.2. Existence and uniqueness of solution	104
3.8. Some particular features of the system of integral equations for the three-part impedance polygon and their consequences	105
3.8.1. Decoupling in the case of unsymmetric three-part impedance planes	105
3.8.2. Decoupling for symmetric three-parts polygons ($\Phi_b = \Phi_a$ and $\sin \theta_+ = \sin \theta_-$)	106
3.8.3. Partial inversion and new kernels for small $k\Delta$	107
3.8.4. Asymptotics for large $k\Delta$	108
3.9. Exact first order expressions for a small complex cavity in a step, when $\Phi_b = -\Phi_a$ and $\sin \theta_+ = \sin \theta_-$	111
3.9.1. Impedance boundary conditions and elementary equations for spectral functions	111
3.9.2. Reduction of far field function when $\sin \theta_+ = \sin \theta_-$ and $\Phi_a = -\Phi_b$	114
3.9.3. Exact first-order expressions for f_1 and for the far field function	115
3.9.4. Exact first-order expression for the diffracted field	116
3.10. References	117
3.A. Appendix A: About $\Psi_{+-}(\alpha, \Phi)$ and the solution for an impedance wedge	119
3.A.1. $\Psi_{+-}(\alpha)$ in passive case	119
3.A.2. Some miscellaneous properties in general case (passive or active)	121
3.A.3. The solution for the diffraction by a wedge with passive or active impedances	122
3.B. Appendix B: principle of semi-inversion for our system of integral equations	122
3.C. Appendix C: uniqueness of fields when impedance boundary conditions on piecewise regular geometry	127
3.C.1. First Green theorem for uniqueness: nullity of field outside S' when $ \arg(ik) < \pi/2$, and of field and its normal derivative on regular parts of S' when $ \arg(ik) = \pi/2$	127
3.C.2. Uniqueness for $ \arg(ik) \leq \pi/2$ comes from nullity of field and its normal derivative on a regular part of boundaries	129
3.C.3. Some extension in 3D electromagnetism	130
3.D. Appendix D: asymptotics for $\mathcal{R}_a(\alpha)$ and $\mathcal{R}_b(\alpha)$ taking account of complex poles	131

3.E. Appendix E: the scattering diagram from the solutions of the integral equations	134
3.F. Appendix F: approximated second-order ameliorations for small cavity	135
3.F.1. Amelioration of equivalent cavity impedance $\sin \theta'_1$	135
3.F.2. A change of M_u	136
Chapter 4. Advanced Properties of Spectral Functions in Frequency and Time Domains for Diffraction by a Wedge-shaped Region	137
4.1. Introduction	137
4.2. Basic properties of spectral function in Sommerfeld–Maliuzhinets representation	138
4.2.1. Sommerfeld–Maliuzhinets representation for scattering by a wedge-shaped sector	138
4.2.2. Basic properties of total field u and spectral function f	141
4.2.3. Higher order expressions with Sommerfeld–Maliuzhinets integrals	143
4.3. Spectral function attached to radiation of a single face and properties	145
4.3.1. Basic integral expression of radiation of one face	145
4.3.2. Spectral function associated with $H_0^{(2)}(kR)$	146
4.3.3. Expression of spectral function associated with u_{\pm} and properties	147
4.4. Far-field radiation of one face and single-face expression of spectral function f	148
4.4.1. The spectral function f derived from fields on a semi-straight line and properties	148
4.4.2. The spectral function f derived from fields on a piecewise smooth semi-line	150
4.5. Expression of f from the far field function F and its consequences	152
4.5.1. Integral expression of f relatively to F	152
4.5.2. Equations on f from functional equations on F and consequences	153
4.5.3. Expression of f from F with a shift of origin	154
4.6. The spectral function $f_0(\alpha)$ for the diffraction by a wedge, with passive or active impedance faces, and an illumination normal to the edge	156
4.6.1. Definition of $f_0(\alpha)$ in general case, with passive or active faces	156
4.6.2. General exact derivation of $\Psi(\alpha) = \Psi^+(\alpha)\Psi^-(\alpha)$	157
4.6.3. Complete asymptotic expressions of $\frac{\Psi^{\pm'}}{\Psi^{\mp}}$ and Ψ^{\pm}	162

4.6.4. Links between Ψ^\pm for passive and active impedances, and positions of poles	165
4.6.5. Determination of $P_n(\sin(\mu\alpha))$ for passive or active faces	166
4.7. Analysis for a wedge at skew incidence (2D1/2) and associated special functions	167
4.7.1. Our solution given in [9]	168
4.7.2. The relation with the solution given by Lyalinov and Zhu	171
4.7.3. Ψ_{an} in terms of Ψ_Φ functions and expressions of Ψ_Φ	173
4.7.4. Expressions for the Bobrovnikov–Fisanov function χ_Φ	177
4.8. Explicitly causal expression in time domain for a dispersive wedge-shaped region	179
4.8.1. Basic elements on causality and integral expressions	179
4.8.2. Properties of spectral function f and trajectories of poles	181
4.8.3. Elementary transform \mathcal{F}_A^\pm of f and its analytical continuation $\mathcal{F}_{a,A}^\pm$	182
4.8.4. Fourier transform of f and vanishing property due to causality	186
4.8.5. Explicitly causal expression of fields in time domain with $\mathcal{F}_{a,A \rightarrow \infty}$	187
4.9. References	189
4.A. Appendix A: analysis of the solution of $s(\alpha \pm \Phi) - \varepsilon s(-\alpha \pm \Phi) = S^\pm(\alpha)$	191
4.B. Appendix B: analysis of the reflection coefficient attached to a multilayered face of a wedge	193
4.C. Appendix C: some miscellaneous properties of spectral functions for an impedance wedge with passive impedance faces	194
4.C.1. The spectral function $f_0(\alpha)$ for a wedge with straight passive faces	194
4.C.2. Some miscellaneous results as $\Phi = \pi/2$ and $\operatorname{Re}(\sin \theta^\pm) \geq 0$	195
4.D. Appendix D: functional equations of high order and behavior of fields at the edge	196
4.D.1. Combinations of derivatives at the edge for the c_n^\pm	197
4.D.2. Combinations of derivatives at the edge isolating $\frac{\partial^m}{\partial(i k \rho)^m} \frac{\partial u}{i k \rho \partial \varphi}$	199
4.E. Appendix E: on the poles of $(\arctan \frac{n}{l})'$ for the evaluation of v_{an}	200
Chapter 5. General Integral Identities for Bianisotropic Media and Related Equations, Properties and Coupling Expressions	201
5.1. Introduction	201
5.2. General integral identities for piecewise continuous media with finite losses	202
5.2.1. Bianisotropic media and adjoint characteristics	202
5.2.2. Basic general properties of fields	203

5.2.3. Initial surface and volume integral identities for piecewise continuous media	204
5.3. Generalized reciprocity theorem and associated fields properties	205
5.3.1. Property 1: principle of equivalence of surface sources enclosing the domain of a volume source in open space	206
5.3.2. Property 2: generalized reciprocity theorem in open space and generalized reciprocity principle	207
5.3.3. Uniqueness properties	207
5.3.4. Property 5: vanishing surface integral equation	209
5.4. Generalized reciprocity principle and dyadic tensors fields	211
5.4.1. Dirac sources in open space and reciprocity principle	211
5.4.2. Generalized reciprocity for co-dipolar dyadic tensors	212
5.4.3. Generalized reciprocity for cross-dipolar dyadic tensors	212
5.4.4. Dyadic tensorial integral expressions of fields and reciprocity	213
5.5. Fields influenced by a perturbation A of a bianisotropic object	213
5.5.1. Definitions of fields	213
5.5.2. Definition of primary sources	214
5.5.3. Integral equalities for (E_{1S}, H_{1S}) and (E_{1S}^0, H_{1S}^0) on S	214
5.5.4. Integral expressions for (E_{1S}, H_{1S}) from fields known on S	215
5.5.5. Definition of influence tensors from $E_{1S} - E_0$ and $H_{1S} - H_0$	219
5.6. Fields influenced by coupling between two scatterers A and A'	219
5.6.1. Definitions of first-order coupling terms $E_{S,S'}$ and $H_{S,S'}$	220
5.6.2. Expressions of $(E_{S,S'}, H_{S,S'})$ for $M'_1 = M_1 = 0$ and for $J'_1 = J_1 = 0$	221
5.6.3. First-order coupling tensors $[E_{S,S'}^{c(m)}(r_1, r'_1)]$ and $[H_{S,S'}^{c(m)}(r_1, r'_1)]$	225
5.6.4. Reciprocity relations for coupling tensors	226
5.7. Application to the numerical removal of couplings	228
5.7.1. Expression of the total field	228
5.7.2. A particular surface radiation integral, its properties and its relation with coupling	228
5.7.3. Efficient determination of the radiation of A without A'	229
5.8. Numerical results	230
5.9. References	233
5.A. Appendix A: on some identities for conjugate fields	233
5.B. Appendix B: on some identities for surface integrals	234
5.B.1. On the FP of a surface integral expression	234
5.B.2. On the principal value (<i>pv</i>) of a surface integral	236
5.B.3. On properties of the surface divergence operator Div	238
5.C. Appendix C: note on distributions and some integrals expressions	239
5.C.1. Elementary equalities for distributions	239

5.C.2. Some integrals concerning the indicator function $1_{\Omega'}(r)$ up to $\partial\Omega'$	239
5.D. Appendix D: integral expressions and equations when S is piecewise smooth	242
5.D.1. General integral expressions for piecewise smooth S	242
5.D.2. Generalities when the domain $\Omega \equiv R^3 \setminus \Omega_S$ holds the isotropic free space	243
5.D.3. Integrals expressions in free space and discontinuities	245
5.D.4. Integral equations for impedance boundary conditions on S	246
5.D.5. Integral expressions when S is a perfectly conducting surface in free space	248
Chapter 6. Exact and Asymptotic Reductions of Surface Radiation	
Integrals with Complex Exponential Arguments to	
Efficient Contour Integrals	251
6.1. Introduction	251
6.2. Formulation	252
6.3. A combination of two fundamental ways for an efficient reduction to a contour integral	254
6.3.1. First way	255
6.3.2. Second way	261
6.3.3. Why these distinct ways are complementary: a practical example	264
6.4. Reduction to contour integrals in case (a) for a pure dipolar illumination of a large perfectly conducting flat plate	267
6.4.1. Basic expansion of kernels in general radiation surface integral expressions	267
6.4.2. Exact reduction to contour integrals when S is a flat plate S' perfectly conducting	269
6.5. Reduction to contour integrals in case (a) for a weighted dipolar illumination of a large perfectly conducting flat plate	273
6.5.1. Integral expressions for a weighted dipolar illumination	273
6.5.2. Reduction to contour integrals for even/odd weight functions $f_{e,h}$	274
6.6. Definition of complex scaling in case (b) and elementary exact surface integral reduction	278
6.6.1. Transformation of the polynomial argument \mathcal{R}^2	279
6.6.2. Reduction of the surface integral to a contour integral	282
6.7. Complex scaling and extended cases of applications for exact reductions	289
6.7.1. Exact reduction when $g \equiv 1$	289
6.7.2. Exact reduction when $g(\mathbf{r}')$ is a polynomial of coordinates	290

6.8. Complex scaling and applications to asymptotic reductions	292
6.8.1. The case with $f(\mathcal{R}^2) = e^{\mathcal{R}^2}$ and semi-asymptotic reduction	292
6.8.2. The case with $f(\mathcal{R}^2) = \frac{e^{2\sqrt{\mathcal{R}^2}}}{\sqrt{\mathcal{R}^2}}$ and asymptotic reduction for curved plate	292
6.9. Reduction in case (c) for surface radiation integrals on curved plates	295
6.9.1. Reduction to a contour integral for curved plates when $f_h \equiv 0$	296
6.9.2. Reduction to a contour integral for curved plates when $f_e \equiv 0$	298
6.10. Miscellaneous results	299
6.11. Numerical results	302
6.11.1. Exact contour reduction of a surface integral whose integrand has an exponential term with a quadratic argument	302
6.11.2. Asymptotic contour reduction of near-field radiation surface integral in physical optics for a large plate illuminated by a point source with spherical pattern	304
6.12. References	317
6.A. Appendix A: primitives for theorems 6.5 and 6.6, and properties	319
6.A.1. Primitives for the theorem 6.5	319
6.A.2. Primitives for the theorem 6.6	320
6.A.3. Expressions and properties of successive primitives of $\frac{e^{-v\tilde{R}}}{(\tilde{R})^n}$	320
6.B. Appendix B: reduction of physical optics surface integrals to contour integrals in bistatic case and quadratic approximation of exponential arguments	322
6.B.1. Physical optics surface integrals and asymptotics	322
6.B.2. An asymptotic development of \tilde{R}	324
6.B.3. Quadratic approximation of exponential arguments and reduction of surface integrals to contour ones	324
6.C. Appendix C: reduction of physical optics radiation integrals in monostatic case for an imperfectly conducting flat plate	325
6.C.1. Physical optics fields and reduction for an electric dipolar source	326
6.C.2. Physical optics fields and reduction for a magnetic dipolar source	327
6.D. Appendix D: particular uses of asymptotics with theorems 6.1 and 6.2	329
6.D.1. A particular use of lemma 6.2 with theorem 6.1	330
6.D.2. A particular use of lemma 6.2 with theorem 6.2	331
6.E. Appendix E: complements on the use of proposition 6.2 for $s(\mathbf{r}') = g(r')f(\mathcal{R}^2)$, when we have $f(\mathcal{R}^2) = e^{\mathcal{R}^2}$ and $g(r')$ is a smooth analytical function on S'	332

6.F. Appendix F: complements on theorem 6.19 for some complicated case	334
6.G. Appendix G: some equalities in relation with proposition 6.11	335
Index	337