
Contents

Acknowledgments	ix
Introduction	xi
Chapter 1. Nonlinear Corrector for CFD	1
1.1. Introduction	1
1.1.1. Linear correction	3
1.1.2. Nonlinear correction	4
1.2. Two correctors for the Poisson problem	5
1.2.1. Notations	5
1.2.2. A priori corrector for the PDE solution	6
1.2.3. Finer-grid DC corrector for the PDE solution	8
1.3. RANS equations	9
1.3.1. Vector form of the RANS system	9
1.3.2. Formal discretization	10
1.3.3. Notations for discretization	11
1.4. Nonlinear functional correction	13
1.4.1. Finite volume nonlinear corrector	13
1.4.2. Finite element corrector	15
1.5. Example: supersonic flow	17
1.6. Concluding remarks	18
1.7. Notes	20
Chapter 2. Multi-scale Adaptation for Unsteady Flows	21
2.1. Introduction	21
2.2. Mesh adaptation efficiency	23
2.2.1. Regular and singular unsteady model	23
2.2.2. Representativity of the spatial interpolation error	24
2.3. Transient fixed-point mesh adaptation scheme	25

2.3.1. Size of subintervals in a mesh convergence	28
2.3.2. Mesh adaptation for unsteady Euler/Navier–Stokes equations with thickened interface	29
2.3.3. Convergent transient fixed-point	33
2.4. 2D bi-fluid example	33
2.5. Example: impact of a 3D water column on a obstacle	35
2.6. Conclusion	39
2.7. Appendix: remarks about the adaptation of the time step	39
2.8. Notes	41
Chapter 3. Multi-rate Time Advancing	43
3.1. Introduction	43
3.2. Multi-rate time advancing by volume agglomeration	45
3.2.1. Finite volume Navier–Stokes	45
3.2.2. Inner and outer zones	46
3.2.3. MR time advancing	47
3.3. Elements of analysis	49
3.3.1. Stability	49
3.3.2. Accuracy	50
3.3.3. Efficiency	51
3.3.4. Toward many rates	52
3.3.5. Impact of our MR complexity on mesh adaption	52
3.3.6. Parallelism	53
3.4. Applications	55
3.4.1. Circular cylinder at very high Reynolds number	55
3.4.2. Mesh adaption for a contact discontinuity	58
3.5. Conclusion	59
3.6. Notes	60
Chapter 4. Goal-Oriented Adaptation for Inviscid Steady Flows	65
4.1. Introduction	65
4.1.1. What to do with this estimate?	67
4.1.2. Adjoint- L^1 approach	68
4.1.3. Outline	69
4.2. A more accurate nonlinear error analysis	69
4.2.1. Assumptions and definitions	69
4.2.2. A priori estimation	70
4.3. The case of the steady Euler equations	72
4.3.1. Variational analysis	72
4.3.2. Approximation error estimation	73
4.4. Error model minimization	74
4.5. Adaptative strategy	76
4.5.1. Adjoint solver	77

4.5.2. Optimal goal-oriented discrete metric	77
4.5.3. Controlled mesh regeneration	79
4.6. Numerical outputs	79
4.6.1. High-fidelity pressure prediction of an aircraft	79
4.7. Conclusion	82
4.8. Notes	82
Chapter 5. Goal-Oriented Adaptation for Viscous Steady Flows	85
5.1. Introduction	85
5.2. Case of an elliptic problem	86
5.2.1. A priori finite-element analysis (first estimate)	86
5.2.2. Goal-oriented adaptation according to lemma 5.1	89
5.2.3. Goal-oriented adaptation according to a second estimate	91
5.3. Error analysis for Navier–Stokes problem	92
5.3.1. Mesh adaptation problem statement	92
5.3.2. Linearized error system	93
5.3.3. First estimate for Navier–Stokes problem	94
5.3.4. Second estimate for Navier–Stokes problem	98
5.3.5. Optimal goal-oriented continuous mesh	101
5.4. From theory to practice	101
5.4.1. Computation of the optimal continuous mesh	103
5.5. An example of application to a turbulent flow	103
5.6. Conclusion	107
5.7. Notes	109
Chapter 6. Norm-Oriented Formulations	111
6.1. Introduction	111
6.2. A summary of previous analyses	114
6.2.1. Feature-based adaptation by interpolation error optimization	114
6.2.2. Implicit a priori error estimate and corrector	115
6.2.3. Goal-oriented analysis	116
6.3. Norm-oriented approach	118
6.4. Numerical elliptic examples	119
6.4.1. Numerical features	119
6.4.2. 2D boundary layer	122
6.4.3. Poisson problem with discontinuous coefficient	123
6.5. Application to flows	126
6.5.1. A comparison feature-oriented/norm	127
6.5.2. Application to a viscous flow	129
6.6. Conclusion	130
6.7. Notes	131

Chapter 7. Goal-Oriented Adaptation for Unsteady Flows	133
7.1. Introduction	133
7.2. Formal error analysis	134
7.3. Unsteady Euler models	135
7.3.1. Continuous state system and finite volume formulation	135
7.3.2. Continuous adjoint system and discretization	137
7.3.3. Impact of the adjoint: numerical example	141
7.4. Optimal unsteady adjoint-based metric	142
7.4.1. Error analysis for the unsteady Euler model	142
7.4.2. Continuous error model	144
7.4.3. Spatial minimization for a fixed t	146
7.4.4. Temporal minimization	146
7.4.5. Temporal minimization for time sub-intervals	150
7.5. Theoretical mesh convergence analysis	155
7.5.1. Smooth flow fields	155
7.6. From theory to practice	157
7.6.1. Choice of the GO metric	158
7.6.2. Global fixed-point mesh adaptation algorithm	158
7.6.3. Computing the GO metric	161
7.7. Numerical experiments	161
7.7.1. 2D Acoustic wave propagation	161
7.7.2. 3D blast wave propagation	163
7.8. Conclusion	165
7.9. Notes	166
Chapter 8. Third-Order Unsteady Adaptation	167
8.1. Introduction	167
8.2. Higher order interpolation and reconstruction	168
8.3. CENO approximation for the 2D Euler equations	170
8.3.1. Model	170
8.3.2. CENO formulation	171
8.3.3. Vertex-centered low dissipation CENO2	174
8.4. Error analysis	175
8.5. Metric-based error estimate	178
8.6. Optimal metric	179
8.7. From theory to practical application	182
8.8. A numerical example: acoustic wave	183
8.9. Conclusion	186
8.10. Notes	186
References	189
Index	199
Summary of Volume 1	201