Contents

Introduction ix

Chapter 1. On Stability of Discrete and Asymptotically Continuous Systems 1

1.1. Stability – general concepts 1
1.2. Buckling of discrete conservative systems – Hencky's column under conservative axial load 18
1.3. Buckling of discrete non-conservative systems – Hencky's column under follower axial load 28
1.4. Stability under kinematic constraints 37
1.5. Conclusion 49
1.6. References 50

Chapter 2. Second-order Work Criterion and Stability in the Small 57

2.1. Some generalities in continuum mechanics for material systems 57
2.2. The link between the kinetic energy and the second-order work 60
2.3. The second-order work in Eulerian formalism 65
2.4. The second-order work on the material point scale 67
2.5. Conclusion 68

Chapter 3. Mixed Perturbations and Second-order Work Criterion 69

3.1. Introduction 69
3.2. From a quasi-static to a dynamical regime 70

3.2.1. Existence of multiple equilibrium configurations 70
3.2.2. Stability of equilibrium configurations 74
3.2.3. Spectral analysis of tensors K and K^s 79
3.3. The case of discrete systems 80
3.3.1. General framework 80
3.3.2. The constrained system 82
3.4. Application to the generalized Ziegler column problem 88
3.4.1. The generalized Ziegler column problem 88
3.4.2. Instability problem with constraints 94
3.5. Concluding remarks 99
3.6. References 100

Chapter 4. Divergence Kinematic Structural Stability 103

4.1. Introduction 103
4.2. KISS issue 105
4.3. The paradigmatic case of the 2 dof Ziegler system 109
4.4. Algebraic approach 112
4.4.1. Schur's complement formula 112
4.4.2. Case of one constraint: $r = 1$ 113
4.4.3. Case of any set of constraints: $1 \leq r \leq n-1$ 113
4.5. Variational approach 114
4.5.1. Variational and minimizing formulations 114
4.5.2. Constraints and quadratic forms *a, b, q*: elimination of Lagrange multipliers 116
4.5.3. Divergence KISS issue 118
4.6. Geometric approach 119
4.7. Coming back to Lyapunov's and Hill's stabilities 121
4.8. References 122

Chapter 5. Flutter Kinematic Structural Stability 125

5.1. Introduction 125
5.1.1. Flutter stability and flutter KISS formulations 126
5.2. Grassmannian and Stiefel manifolds 128
5.3. Case $n = 3$ and $m = 2$ 131
5.3.1. Geometric considerations and preliminary calculations 132
5.3.2. Sufficient conditions 137
5.3.3. Necessary and sufficient conditions: calculations in $\mathbb{S}_{2,3}(\mathbb{R})$ 139
5.3.4. Necessary and sufficient conditions: calculations in $\mathbb{S}^2$ 143
5.3.5. Summary of the results 148

5.4. Partial flutter KISS: examples 148
5.4.1. Mechanical consequences 148
5.4.2. Examples 149
5.4.3. M = I_3 152
5.4.4. Uniform mass distribution 154
5.5. References 155

Chapter 6. Geometric Degree of Non-conservativity 157

6.1. Introduction 157
6.2. Modeling and calculation of the GDNC: examples 158
6.2.1. Calculation of the GDNC 158
6.2.2. Examples 161
6.3. Calculation of $\mathfrak{C}_g$ 173
6.3.1. The GDNC and the exterior calculus 174
6.3.2. Set of solutions 176
6.3.3. An example 177
6.4. Extension to the nonlinear framework 181
6.4.1. Nonlinear issue and notations 182
6.4.2. Link with the linear framework 184
6.5. Solution of the nonlinear problem 185
6.5.1. The solution 185
6.5.2. Effectiveness of the solution 187
6.5.3. Set of solutions 188
6.5.4. The example 188
6.6. Duality KISS-GDNC 194
6.7. References 195

Chapter 7. Buckling of Granular Systems with Shear Interactions: Discrete versus Continuum Approaches 199

7.1. Introduction – instabilities of granular systems 199
7.2. Shear granular system – a discrete approach 201
7.3. Buckling of granular system – exact solution 206
7.4. Shear granular system – a continuous approach 211
7.5. Conclusion 218
7.6. References 218

Chapter 8. Continuous Divergence KISS 223

8.1. Introduction 223
8.2. Description of the problem 224
8.2.1. Strong formulation of divergence stability of the Beck column and its Ziegler system counterpart 224

8.2.2. Usual weak formulation of divergence stability of the Beck column and its Ziegler system counterpart 226
8.2.3. Results in a finite dimension space and position of the problem 228
8.3. Fundamental Spaces – topological aspects. Variational formulation of the problem 230
8.3.1. Vector spaces and operator *A(P)*: usual aspects 230
8.3.2. Kinematic constraints 232
8.3.3. An alternative formulation involving only the Hilbert space $\mathcal{V}$ equipped with $(.|.)_{\mathcal{V},2}$: solution of issue 1 234
8.3.4. Variational formulation for the KISS problem 235
8.3.5. Geometric solution of the KISS issue: compression of an operator 236
8.4. Solution for the kernel of the operators $A_s(P)$ and $\tilde{A}_s(P)$ 239
8.4.1. Calculation of $A_s(P)$ 240
8.4.2. Calculation of $\tilde{A}_s(P)$ 242
8.4.3. Calculation of the critical load P_2^* 243
8.4.4. Calculation of ker $A_s(P_2^*)$ = ker $\tilde{A}_s(P_2^*)$ 244
8.4.5. Calculation of the optimal destabilizing constraint 244
8.5. $P_1^* = P_2^*$ 246
8.6. Stability issues 249
8.7. Appendices 252
8.7.1. Equivalence between different forms of kinematic constraints 252
8.7.2. Eigenvalues problem for $\tilde{A}_s(P)$ 253
8.7.3. Figures 257
8.8. References 258

Index 263