## Contents

**Introduction** .......................................................... xi

**Chapter 1. The Thermal Worm Model to Represent Entropy–Exergy Duality** ................................................................. 1

1.1. A fractal and diffusive approach to entropy and exergy ................. 1
   1.1.1. The filamentary thermal worm model of exergy .................. 1
   1.1.2. The geometrical Carnot factor and the temperature of a filamentary worm ......................................................... 3
   1.1.3. The T-like filamentary worm ...................................... 6
1.2. A granular model of energy: toward the entropy and the exergy of a curve ................................................................. 7
   1.2.1. The model of granular energy: the concept of ergon ............ 7
   1.2.2. Exergy and anergy of an ergon: the entropic angle of an energy ................................................................. 7
   1.2.3. An elementary ergon discharged in a field of temperature T: the thermal puff time of energy ................................. 9
   1.2.4. The granular energy model applied to the hydraulic analogy of Lazare and Sadi Carnot: entropy and action of energy .... 11
   1.2.5. Exergy and anergy of a curve ...................................... 15
   1.2.6. Examples of Carnot factor of some curves ....................... 18
1.3. The thermal worm model of entropy–exergy duality ....................... 19
   1.3.1. Entropic skins to describe irreversibilities and entropy–exergy duality ................................................................. 19
   1.3.2. The 2D worm model .................................................. 21
   1.3.3. The 3D worm model: the coefficient of entropic dispersion .... 22
   1.3.4. Entropic structure of a steady-state heat flux .................... 23
   1.3.5. 2D worm entropic dispersion of a steady heat flux ............. 24
   1.3.6. 3D worm entropic dispersion of a steady heat flux ............. 25
1.4. The 2D worm model .................................................. 26
   1.4.1. The isothermal 2D worm ........................................... 26
1.4.2. Exergy destruction between two 2D worms .................................. 27
1.4.3. The non-isothermal 2D worm ..................................................... 28
1.4.4. The 2D worm of a linear profile of temperature: method 1 ........... 31
1.4.5. 2D worm displaying a linear temperature profile: method 2 ....... 35
1.5. The 3D thermal worm-like model ................................................ 36
1.5.1. The isothermal 3D worm model .............................................. 36
1.5.2. The entropic angle of energy ................................................... 39
1.5.3. The non-isothermal 3D worm model ....................................... 40
1.5.4. Table to recapitulate .............................................................. 43
1.5.5. A link with the phenomenon of intermittency in fully developed turbulence ................................................................. 44
1.5.6. Longitudinal diffusion and lateral diffusion in the worm-like model ................................................................. 46

Chapter 2. Black Hole Entropy and the Thermal Worm Model .......... 49

2.1. Entropy of a black hole: the Bekenstein–Hawking temperature .... 49
2.1.1. Introduction: a geometric entropy for black holes .................. 49
2.1.2. Gravitational and quantum diffusivities: longitudinal and lateral diffusivities of a black hole? .................. 52
2.1.3. The existence of an absolute minimum temperature which is not 0 ................................................................. 54
2.2. The thermal worm model of black holes .................................. 56
2.2.1. Entropic thermal worm representation of a black hole ......... 56
2.2.2. Temperature of the worm and temperature of the black hole .... 58
2.2.3. The thickness of the horizon of a black hole ............... 59
2.2.4. The quantum variation of the temperature in a black hole ....... 60
2.3. Carnot representation of black holes .......................................... 62
2.3.1. Black hole as a reversible power cycle .......................... 62
2.3.2. Black hole as a reversible refrigeration cycle .......... 62
2.3.3. The quantum interaction velocity ............................... 63
2.3.4. Relaxation time of thermal inhomogeneities: the “thermal puff time” of energy in a black hole ........ 65
2.3.5. From Planck’s constant and Boltzmann’s constant toward Brillouin’s constant $b = h/k$ ............. 67
2.3.6. Black hole physics: a deep and fascinating representation of the finiteness of our world .................. 68


3.1. Intermittency of black-body radiation ................................. 71
3.1.1. Black-body radiation: Wien’s law, Rayleigh–Jeans’ law and Planck’s law ......................................................... 71
3.1.2. The spectral volume fraction and the intermittency of radiation 73
3.1.3. A simple derivation of $u(\nu) = \frac{8\pi \nu^2}{c^2} E_\nu$, using an analogy with fully developed turbulence ........................................ 75
3.1.4. A simple understanding of the importance of the ratio $\nu/T$ in the black-body problem .................................................. 76
3.1.5. The interacting length and interaction time of a photon and its scale-entropy ..................................................... 77
3.1.6. The interacting length of a fermion and its scale-entropy .......... 81
3.1.7. Summary of the chapter ................................................. 82
3.2. Generalized RJ law based on a scale-dependent fractal geometry ........ 82
3.2.1. Radiation as stationary waves in a box: the Rayleigh description ................................................................. 83
3.2.2. First case: uniform distribution of modes in phase-space – the RJ theory ...................................................... 84
3.2.3. Second case: fractal distribution of modes in phase space ........ 86
3.2.4. General case: the fractal dimension is mode dependent ............ 87
3.2.5. Conclusion ................................................................. 88
3.3. Fluctuations and energy dispersion in black-body radiation ................. 88
3.3.1. Planck’s derivation using the second-order derivative of entropy: the “lucky guess” ....................................................... 88
3.3.2. Planck’s entropy .................................................................. 91
3.3.3. Planck’s entropy, Planck’s counting of complexions and Boltzmann’s theory ...................................................... 93
3.3.4. Planck’s entropy and the scale-entropy of a radiative field .......... 97
3.3.5. Equivalent chemical potential of a Boltzmann distribution .......... 99
3.3.6. Graphical method to represent Boltzmann’s counting ............... 100
3.3.7. What is the fundamental status of the second order derivative of entropy relative to energy? Variance and the dispersion factor of radiative energy .................. 103
3.3.8. The fluctuations and the dispersion of energy ......................... 108
3.3.9. The dispersion factor interpreted by the worm model ............. 111
3.3.10. An heuristic thermodynamical uncertainty relation from the thermal worm model ............................................. 112
3.4. A scale-entropy diffusion equation for black-body radiation ............. 114
3.4.1. Spacions and scale-entropy .............................................. 114
3.4.2. Scale-entropy diffusion equation .......................................... 116
3.4.3. Pure truncated fractal case and parabolic scaling ....................... 119
3.4.4. Exponential scaling: the scale-entropy sink is a fraction of the entropy production of a photon ......................... 119
3.4.5. Summary ......................................................................... 122
3.5. Spectral fractal dimensions and scale-entropy of black-body radiation .......................................................... 122
### Chapter 4. Non-extensive Thermodynamics, Fractal Geometry and Scale-entropy

4.1. Tsallis entropy in non-extensive thermostatistics
4.2. Two physical systems leading to Tsallis entropy:
   - Work produced by the isothermal growth of a fractal volume
   - Mass decay of a fractal system
4.3. Non-extensive thermostatistics, scale-dependent fractality and Kaniadakis entropy
   - Decaying process of a fractal system submitted to internal cohesion pressure
   - The Kaniadakis $\kappa$-statistics and Kaniadakis entropy
4.3.3. Conclusion on non-extensive thermodynamics and fractal geometry and scale-entropy

### Chapter 5. Finite Physical Dimensions Thermodynamics

5.1. A brief history of finite physical dimensions thermodynamics
5.2. Transfer phenomena by FPDT
   - Series model of insulation: thermal resistances in series
   - Parallel model of insulation: thermal resistances in parallel
5.2.3. Generalization
5.2.4. Partial conclusion
5.3. Energy conversion by FPDT
   - Carnot cycle and thermodynamics of equilibrium
   - The non-adiabatic endoreversible Carnot engine
   - The adiabatic endoreversible Carnot engine
   - The adiabatic non-reversible Carnot engine
   - The non-reversible Carnot engine with thermal losses
5.3.6. Generalization of previous results
5.4. Extension to complex systems: cascades of endoreversible Carnot engines
   - Cascade of power Carnot engines
5.4.2. Thermodynamic model in finite dimensions .......................... 170
5.4.3. Optimization of the cascade ............................................. 171
5.4.4. Cascade with N endoreversible machines ............................ 173
5.5. Time dynamics of Carnot engines ........................................ 174
5.5.1. Reversible thermal transfer between source and sink .............. 174
5.5.2. Finite transfer between source and irreversible engine .......... 176
5.6. Conclusions on FPDT ...................................................... 179

Chapter 6. A Scale-Dependent Fractal and Intermittent Structure to Describe Chemical Potential and Matter Diffusion ............................. 181

6.1. Defining and quantifying the diffusion of matter through chemical potential ................................................................. 181
6.1.1. The chemical potential .................................................. 181
6.1.2. Fundamental equations .................................................. 183
6.1.3. Condition of chemical equilibrium ................................... 184
6.2. Topic scales and scale-entropy of a set of particles .................. 186
6.2.1. Topic volume and topic scales of a particle ....................... 186
6.2.2. Scale-entropy of a set of particles ................................... 188
6.2.3. Scale-entropy of a fractal set ........................................ 190
6.3. Entropy and chemical potential of an ideal gas by Sackur–Tetrode theory ................................................................. 192
6.3.1. Entropy of monoatomic ideal gases using Sackur–Tetrode theory ................................................................. 192
6.3.2. Chemical potential using Sackur–Tetrode theory .................. 195
6.3.3. Chemical potential of a mixture using Sackur–Tetrode theory .. 196
6.4. Entropy of a set of particles described through topic scales and scale-entropy ................................................................. 198
6.4.1. Waving and clustering entropies of a set of particles .......... 198
6.4.2. Application to a two-component mixture: the clustering entropy of a mixture ................................................................. 200
6.4.3. Application to heterogeneous solids: clustering entropy of heterogeneity ................................................................. 203
6.5. Fractal and scale-dependent fractal geometries to interpret and calculate the chemical potential ................................................................. 204
6.5.1. Chemical potential interpreted via scale-entropy, expelling potential and clustering entropy ................................................................. 204
6.5.2. Intermittent and multiscale nature of chemical potential: the fractal case ................................................................. 205
6.6. The intermittency parameter and clustering entropy of particles in the fractal case ................................................................. 206
6.6.1. Clustering entropy of a single i-particle belonging to a fractal set of particles ................................................................. 206
6.6.2. Clustering entropy for the whole ith component (fractal case) .. 208