Introduction

In their daily practice, professional engineers are often confronted with problems involving complex physical phenomena. Whether they have to analyze the behavior of a product or optimize an implementation process, modeling these phenomena often makes it possible to solve these problems:

– by gaining access to the physical magnitudes characterizing the product and the consequences of their modification,

– by highlighting the main parameters of the process and thus its optimal conditions of use.

Physical modeling seems to be the key point when an engineer is thinking of how to solve a scientific problem. Generally speaking, modeling follows these three steps:

1) identifying the essential physical phenomena related to the behavior of a product or process within a given context,

2) defining the mathematical equations (domain of study, partial differential equation(s), boundary conditions and stresses, initial conditions, etc.) governing these phenomena,

3) validating the equations with respect to the necessary data which must be available or accessible by means of measurements as well as the results provided, which must be reliable and reproducible.

The first two points need commenting upon. Firstly, physical modeling has to meet a set objective. It is then obvious that several modeling methods are possible to meet a similar objective. The most relevant model is the simplest one making it possible to meet the set objective. Modeling is first and foremost making (simplifying) assumptions. Some are obvious. For instance, it is unnecessary to take into account heat radiation phenomena in low temperature applications; a mean exchange coefficient, independent of the temperature, will be sufficient. In other cases, it is far more difficult. For example, how can we define the analyzed field and include the behavior of the parts not addressed by means of carefully selected boundary conditions? This is exactly where the professional engineer's contribution comes in. We will come back to this later on.

The last point of this definition is about the model input data and results. The selection of input data (availability, validity, etc.) often guides the users in the choice of their modeling methods. It is unnecessary, for instance, to use very sophisticated metallurgical transformation models if sufficiently accurate data on the processed material are not available. Result reliability and reproducibility guide the users in the choice and use of the model solution method. The aim of this method (numerical or analytical) is to determine a solution of the problem with minimum approximation. It is obvious that an accurate analytical solution will always be given priority; however, the growing complexity of the model often leads today's users towards approximate numerical solutions. Moreover, commercial numerical simulation software often includes the functions and models required for industrial applications, thus making it possible to take full advantage of the progress of information technology (rapid calculation, memory resources, graphic visual aids, etc.). Consider for instance the boundary value problem related to steady state heat exchanges in a solid occupying a limited field (equation [1.8]), which reveals:

– a volumetric heat source Q most often resulting from other physical phenomena such as the Joule effect in conduction heating or induction heating applications. Is it necessary to model these phenomena by adding the corresponding partial differential equations to the heat equation? This is not certain but then we require simple analytical models to evaluate Q;

– a surface density of the heat flux q applied to a portion of the solid boundary, reflecting the heat exchange between this solid and the outside medium. This can be for instance heat radiation towards an infinite medium or a liquid flow into a quenching bath. Here again, is it necessary to include those phenomena in the modeling process with the addition of complementary equations? It depends on the application considered and the feasibility of such an approach. However, if, whatever the reason, fine modeling of these phenomena was not envisaged, other approaches would then be necessary to determine the boundary conditions. These could be, for instance, simplified analytical models;

– a temperature prescribed on the other portion of the solid boundary. This condition is a limit case of the previous condition. As a matter of fact, either the solid is steeped at this spot in a fluid with a very high exchange coefficient so as to prescribe the fluid temperature on the corresponding boundary, or the user has a measurement.

Experimental measurements will frequently be used to determine the missing input data, but these will be obtained by alternative routes as the data required for modeling are often inaccessible to direct measurements. The methods called inverse methods have developed significantly over the last few years. They are able to couple physical modeling with accessible magnitude experimental measurements by adjusting iteratively an input data until the whole set of calculated results are as close as possible to the measurements.

It is tempting to say after these comments that the easiest model to elaborate is one in which all influential physical phenomena are finely modeled. This is partly true; however:

– Are we really in a position to define the mathematical equations governing these phenomena with a sufficient degree of accuracy to make it worthwhile?

– Are we certain that we are able to have the data required to feed these models all the more so that these models will frequently require unusual data on a different scale, which is thus hardly accessible? Using inverse methods may solve this type of difficulty.

– Will it not be often more difficult to interpret the results than with a simpler model given the amount of information to process? Indeed, will it not be necessary to conduct *a posteriori* the analysis which will not have been carried out *a priori* to eliminate insignificant results?

In fact, everything depends on the problem to solve. Moreover at which point of the domain of study considered should we stop? First, we will take into consideration all the symmetries presented by the problem for which the corresponding boundary conditions are written clearly from a mathematical point of view:

– revolution symmetry: the most productive one as it allows us to carry out 3D analysis on a plane model representing a meridian section of the studied structure,

– symmetry to a plane: very commonly used,

– anti-symmetry to a plane: less natural and often forgotten,

– periodicity conditions on a repetitive structure.

The relevance of a 3D model will then be questioned. If it is true that calculation software and computers make it easy to carry out this type of analysis, a 2D (and even 1D) model is often sufficient (in the case of a repetitive structure in one direction for example) and therefore preferable as the analysis and interpretation of results will always be easier with a 2D than with a 3D model. In all other cases, restricting the domain of study will lead to the definition of appropriate boundary conditions.

It is therefore in the interest of the engineers in charge of physical modeling to think in detail about the relevance of their choices and assumptions. It is the price to pay to take maximum advantage of their models. In any case, before reaching the intensive exploitation stage, it is imperative to ensure the model prediction quality by comparing it with the results of one experiment (at least!).

Most often, analytical methods cannot be applied to the solution of the mathematical equations governing a set of physical phenomena, except if major assumptions reducing the modeling validity are made. The analog method takes advantage of the fact that conduction heat exchanges and electrical conduction phenomena are governed by the same equations. It is thus possible to study conduction heat exchanges by means of more easily accessible measurements carried out on a similar electrical device. However, the field of application of this method is far too restrictive. Therefore, numerical techniques whose use is made possible by the performance of today’s computers are used to determine an approximate solution of the set of mathematical equations governing the problem.

The finite difference method which replaces partial derivatives with finite differences at different points of a grid is highly regarded by mechanical engineers. The discrete equation system obtained can also be interpreted as resulting from a complex electrical diagram combining resistances and capacities. This technique is hardly possible with complex geometries.

The finite volume method is also highly regarded by mechanical and thermal engineers. It is based upon a previous division of the geometric domain of study into element volumes. On each element volume, the thermal balance equations are solved. This method is particularly efficient in the case of structured geometries. It is widely used in thermal science and fluid mechanics, but more rarely in other disciplines.

Among the various numerical techniques available today, the finite element method [TOU 81, ZIE 91] is the most widespread owing to:

– its general fields of application (thermal, electromagnetic sciences, solid mechanics, fluid mechanics, etc.),

– its capacity to treat problems with complex geometries,

– its easy implementation.

This can be done as follows:

1) mesh-geometry: the geometric domain to be analyzed, most often resulting from CAD geometric modeling, is divided into a set of element sub-domains (finite elements) interconnected by nodes;

2) solution: the continuous functions sought (the temperature in a heat conduction problem) are replaced with a set of values estimated at the mesh nodes. This approximation, applied to an integral formulation of the problem, leads to a system of equations (linear or non-linear) whose number is equal to the number of values to be estimated;

3) analysis-interpretation of results: at this point, other results can be calculated (for instance, the heat flux density) from those obtained by the direct solution of the equation system. They are then analyzed and interpreted by the user, by means of very efficient graphic visualization resources.

The success of the finite element method is largely due to the significant progress of information technologies, both from a numerical point of view (rapid calculation, memory size available) and from a graphic point of view (3D visual resources). Today a large amount of software makes use of this method. They offer a growing number of functions and are increasingly user-friendly. It should be noted that calculation software displays clear and user-friendly interfaces nowadays and can be used by non-specialist engineers.

However, this apparent facility should not conceal the fact that, whatever the numerical method adopted, the discretization phase impairs the properties of the initial continuous model. Some phenomena present in the continuous model could be erased by the numerical model if we do not pay attention. The meshing phase in the finite element method is therefore very important and it is the user's know-how that will produce a quality approximate solution.

Therefore, the aim of this book is to present the basis and application of the finite element method to the solution of industrial thermal problems. It consists of three parts which the reader may possibly complete by reading a number of books related to this field [COM 94, RED 94, LEW 96, MIN 06].

Part 1, dedicated to the solution of steady state heat conduction problems, introduces the finite element method. Starting with the partial derivative problem and the related boundary conditions, various formulations upon which various discretization methods are based, are presented.

Part 2 extends the field of application of the method to transient state conduction problems, the most common non-linearities and transport phenomena (diffusion convection problems).

The last part, Part 3, deals with coupled problems:

– coupled by boundary conditions: radiation problems, fluid and structure coupling in a piping system,

– including additional state variables: thermometallurgical coupling,

– coupled by partial differential equations: electrothermal coupling, magnetothermal coupling and thermochemical coupling.

This book is a survey of the various thermal problems which professional engineers may have to simulate. The methods presented will allow readers to use in the best way possible a calculation software and design new calculation modules so as to complete their work.