Table of Contents

**PART 1. CONSTRAINT PROGRAMMING AND FOUNDATIONS OF GRAPH THEORY** ........................................ 1

**Introduction to Part 1** ........................................ 3

**Chapter 1. Introduction to Constraint Programming** ................................................................. 5

  1.1. What is a variable? ........................................ 7
  1.2. What is a constraint? .................................... 8
  1.3. What is a global constraint? ............................ 10
  1.4. What is a propagation algorithm? ...................... 11
  1.5. What is a consistency level? ............................ 14
  1.6. What is a constraint solver? ............................ 15
  1.7. Constraint solvers at work ............................ 17
     1.7.1. Importance of modeling .......................... 17
     1.7.2. Importance of heuristics in guiding research ... 20
     1.7.3. Importance of using global constraints .... 21
  1.8. Organization structure ................................. 21

**Chapter 2. Graph Theory and Constraint Programming** ............................................................. 23

  2.1. Modeling graphs with constraint programming ........ 24
Chapter 3. Tree Graph Partitioning

3.1. In undirected graphs .......................... 39
3.2. In directed graphs .......................... 42

PART 2. CHARACTERIZATION OF TREE-BASED
GRAPH PARTITIONING CONSTRAINTS ............... 47

Chapter 4. Tree Constraints in Undirected
Graphs ............................................. 49

4.1. Decomposition ................................. 49
4.2. Definition of constraints ...................... 51
4.3. A filtering algorithm for the proper-forest
constraint ........................................... 56
4.3.1. A solution for the proper-forest
constraint ........................................... 57
4.3.2. Hybrid-consistency for the proper-forest
constraint ........................................... 59
4.3.3. Correction and completion .................. 61
4.3.4. Complexity ................................... 64
4.4. Filtering algorithm for the resource-forest
constraint ............................................. 70
4.4.1. Existence of a solution for the
resource-forest constraint ......................... 70
4.4.2. Hybrid-consistency for the resource-forest
constraint ............................................. 72
4.4.3. Correction and completion .................. 73
### 4.4.4. Complexity .................................. 79
### 4.5. Summary of undirected tree constraints .... 80

#### Chapter 5. Tree Constraints in Directed Graphs .............................. 83

5.1. Decomposition ................................. 83
5.2. Definition of constraints .................... 86
5.3. Filtering algorithm for the tree constraint ... 89
  5.3.1. Existence of a solution for a tree constraint ................................ 89
  5.3.2. General arc-consistency for the tree constraint ............................. 91
  5.3.3. Correction and completion .......................... 93
  5.3.4. Complexity .................................. 96
5.4. Filtering algorithm for the proper-tree constraint .......................... 96
  5.4.1. Limits on the number of proper trees ........ 99
  5.4.2. Existence of a solution for the proper-tree constraint ................ 103
  5.4.3. Filtering algorithm for the proper-tree constraint ........................ 104
  5.4.4. Correction .................................. 109
  5.4.5. Complexity .................................. 112
5.5. Summary of tree constraints in directed and undirected graphs .......... 113

#### Chapter 6. Additional Constraints Linked to Graph Partitioning ................. 117

6.1. Definition of restrictions ..................... 118
6.2. Complexity zoo ................................ 123
  6.2.1. Proper trees ............................... 124
  6.2.2. Precedence constraints .................. 124
  6.2.3. Conditional precedence constraints ....... 126
  6.2.4. Constraints on the interior half-degree of vertices...................... 127
6.2.5. Incomparability constraints ............ 128
6.3. Interaction between the number of trees and
the number of proper trees ................. 129
6.4. Relation of precedence between the vertices of
the graph ................................ 130
   6.4.1. Limitations on the maximum number of
trees ................................... 131
   6.4.2. Filtering linked to a set of precedence
constraints ............................. 132
   6.4.3. Filtering and complexity algorithm ..... 134
6.5. Relation of conditional precedence .......... 137
   6.5.1. Filtering linked to a conditional
precedence set .......................... 138
   6.5.2. Algorithmic and complexity .......... 139
6.6. Relation of incomparability between graph
vertices ............................... 140
   6.6.1. Filtering linked to incomparability
constraints ............................. 141
   6.6.2. Filtering and complexity algorithm ..... 142
6.7. Interactions between precedence and
incomparability constraints .................. 143
   6.7.1. Improving filtering via interactions ... 143
   6.7.2. Deduction of new precedence
constraints ............................. 146
6.8. Constraining the interior half-degree of each
vertex .................................. 148
6.9. Summary ................................ 151

Chapter 7. The Case of Disjoint Paths .......... 153

7.1. Minimum number of paths in acyclic directed
graphs ................................... 156
7.2. Minimum number of paths in any directed
graph .................................... 161
   7.2.1. Estimating the number of paths
partitioning a CFC ....................... 164
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2.2.</td>
<td>Estimating the number of paths between two CFCs</td>
<td>167</td>
</tr>
<tr>
<td>7.3.</td>
<td>A path partitioning constraint</td>
<td>169</td>
</tr>
<tr>
<td>7.4.</td>
<td>Summary</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td><strong>Chapter 8. Implementation of a Tree Constraint</strong></td>
<td>175</td>
</tr>
<tr>
<td>8.1.</td>
<td>Original implementation</td>
<td>176</td>
</tr>
<tr>
<td>8.1.1.</td>
<td>The tree constraint</td>
<td>176</td>
</tr>
<tr>
<td>8.1.2.</td>
<td>The extended-tree constraint</td>
<td>179</td>
</tr>
<tr>
<td>8.1.3.</td>
<td>Limitations of the approach: illustration using the tree constraint</td>
<td>180</td>
</tr>
<tr>
<td>8.2.</td>
<td>Toward a “portable” implementation</td>
<td>181</td>
</tr>
<tr>
<td>8.2.1.</td>
<td>Implementation</td>
<td>183</td>
</tr>
<tr>
<td>8.2.2.</td>
<td>Bench mark</td>
<td>185</td>
</tr>
<tr>
<td>8.2.2.1.</td>
<td>Historic implementation versus adaptable implementation</td>
<td>186</td>
</tr>
<tr>
<td>8.2.2.2.</td>
<td>Evaluation of portability</td>
<td>189</td>
</tr>
<tr>
<td>8.3.</td>
<td>Conclusion</td>
<td>191</td>
</tr>
<tr>
<td></td>
<td><strong>PART 3. IMPLEMENTATION: TASK PLANNING</strong></td>
<td>193</td>
</tr>
<tr>
<td></td>
<td><strong>Introduction to Part 3</strong></td>
<td>195</td>
</tr>
<tr>
<td></td>
<td><strong>Chapter 9. First Model in Constraint Programming</strong></td>
<td>199</td>
</tr>
<tr>
<td>9.1.</td>
<td>Model for the coherence of displacements in space</td>
<td>199</td>
</tr>
<tr>
<td>9.2.</td>
<td>Modeling resource consumption</td>
<td>200</td>
</tr>
<tr>
<td>9.3.</td>
<td>Modeling time windows</td>
<td>201</td>
</tr>
<tr>
<td>9.4.</td>
<td>Modeling coordination constraints between units</td>
<td>202</td>
</tr>
<tr>
<td>9.5.</td>
<td>Limitations of the proposed model</td>
<td>203</td>
</tr>
</tbody>
</table>
Chapter 10. Advanced Model in Constraint Programming ........................................... 205

10.1. Modeling the coherence of displacements in space ........................................ 206
10.2. Modeling resource consumption ............................................................... 208
10.3. Integration of temporal aspects ................................................................. 208
10.4. Propagating time windows ....................................................................... 213
   10.4.1. Interaction with the graph to be partitioned ........................................ 213
   10.4.2. Interaction with the precedence graph .............................................. 215
   10.4.3. Deriving impossible paths ............................................................... 217
   10.4.4. Interaction with the original tree constraint ....................................... 219
   10.4.5. Complexity .................................................................................. 219
   10.4.6. Integration into the mission planning model ....................................... 222

Part 4. CONCLUSION AND FUTURE WORK .................................................... 225

Chapter 11. Conclusion ................................................................................. 227

Chapter 12. Perspectives and Criticisms ....................................................... 231

Bibliography .................................................................................................. 233

Index ............................................................................................................ 239