1.1. Cohesion forces

In a real structure, it is necessary to accept the existence of a system of internal cohesion forces which originate from intermolecular actions and which allow, among other things, the preservation of the initial form of the structure.

In a structure made of a material we shall assume to be elastic[3], let us isolate a particle of matter specified by a *very small sphere* around a point M (Figure 1.1a).

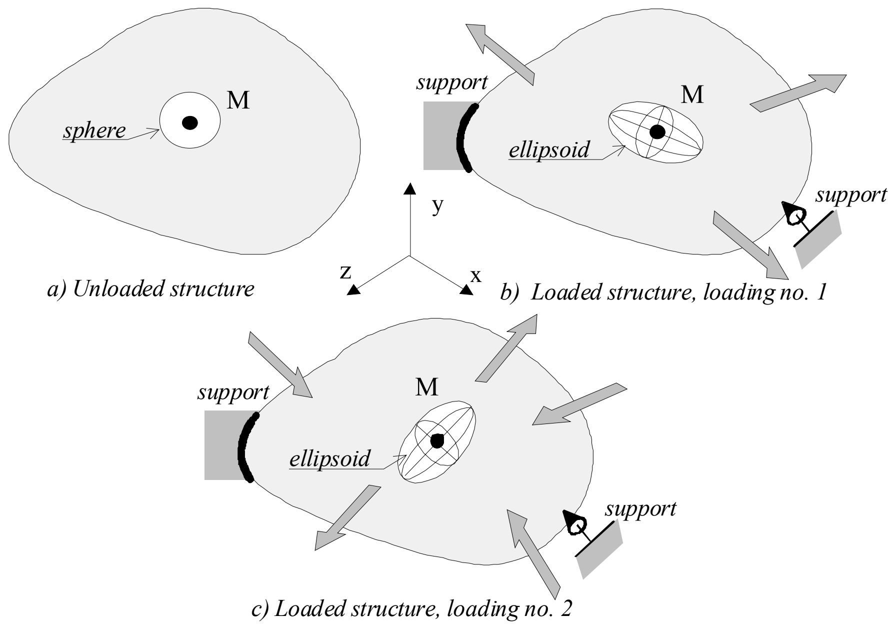
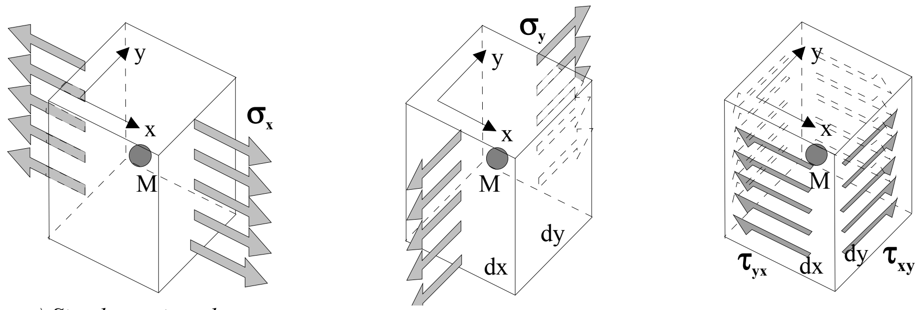


Figure 1.1. *A spherical domain's deformation around point M*

When this structure is loaded, point M undergoes a displacement (Figures 1.1b and 1.1c) which we shall assume to be very small when compared to the dimensions of the structure, so that the latter's shape does not vary perceptibly. It is shown for all materials made up of standard structures, that the small spherical domain around the point M first deforms weakly[4] becoming an ellipsoid. The shape and the orientation of that ellipsoid change not only with the position of point M in

3 We shall return later on (section 1.3) to this notion of elastic material.

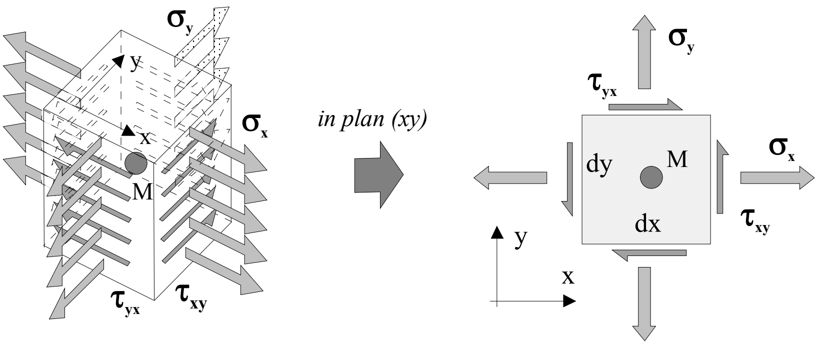
4 The deforming steps (ellipsoid) of Figures 1.1 and 1.3 are greatly exaggerated for standard metal alloys; in reality the variation in the shape is imperceptible as the displacement of all the points such as M is of very small amplitude compared to the dimensions of the structure.



a) Simple tension along x (or respectively compression) *b) Simple tension along y (or respectively compression)* *c) Pure shear*

Figure 1.15. *Components of a plane state of stresses*

The complete plane state of stresses is represented in Figure 1.16 by superposing the three simple states.



a) spatial representation *b) plane representation*

Figure 1.16. *Complete plane state of stresses*

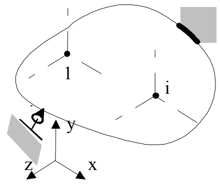
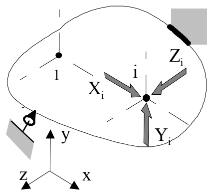
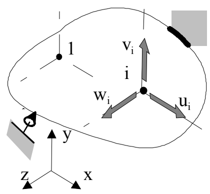
We may regroup the characteristic components in a column matrix[15]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}.$$

15 Note: the terms of this column matrix are not the three components of the same stress vector. Remember that, in fact, $\overrightarrow{C_{(M,\vec{x})}} = \sigma_x \vec{x} + \tau_{xy} \vec{y}$ and $\overrightarrow{C_{(M,\vec{y})}} = \tau_{yx} \vec{x} + \sigma_y \vec{y}$.

2.4.4. *Summary*

All the examples of the loaded structures in sections 2.4.1 to 2.4.3 have general properties that can be summarized in the following table.

structure	Loading and degrees of freedom of a structure
	❒ any geometric structure; ❒ composed of a linear elastic material; ❒ having links with its surroundings; ❒ associated with structural or global coordinates ($\vec{x},\vec{y},\vec{z}$); ❒ having "n" points (i) = [1,…n] within and on its external surface.
loads	
	❒ each point can be loaded by several force components, for example X_i, Y_i, Z_i at point (i); ❒ the load vector is represented as: $\{F\} = \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ ... \\ Z_n \end{Bmatrix}$; ❒ the loads can include moments acting on the small zones surrounding points (i).
degrees of freedom	
	❒ the displacement of every point (i) has as components u_i, v_i, w_i. These are the degrees of freedom (dof); ❒ the dof vector is represented as: $\{d\} = \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ ... \\ w_n \end{Bmatrix}$ ❒ the dof can include rotations of small areas surrounding points (i).
characteristic equations	
behavior equation: $\{d\} = [\alpha] \bullet \{F\}$ *work/potential energy:* $W = \frac{1}{2}\{F\}^T \bullet [\alpha] \bullet \{F\} = E_{pot.}$ $W = \frac{1}{2}\{F\}^T \bullet \{d\} = E_{pot.}$	❒ $[\alpha]$ is the flexibility matrix. It is square and symmetric; ❒ it has the same number of lines and columns as the degrees of freedom; ❒ there is a dual association between a force component and its dof: – *geometric* association: same geometric direction for the force and its dof; – *energy* association: the work developed results from the product of the forces and their associated dof.

[2.91]

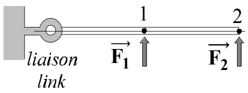
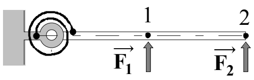
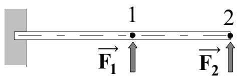
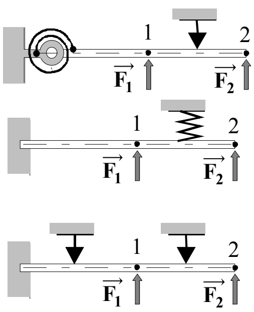
same structure, same nodes (1) and (2), same loads F_1 and F_2, different linking conditions		
	❒ incomplete positioning of the structure before loading: "*hypostatic*" positioning.	the structure is not "properly linked" ❒ $[\alpha]$ cannot be defined ❒ equation $\{d\}=[\alpha]\bullet\{F\}$ is not defined
	❒ complete positioning of the structure before loading is called "*isostatic*" positioning *(the equations of equilibrium are sufficient to obtain the link forces on the structure under loading).*	❒ the structure is "properly linked" ❒ the flexibility matrix $[\alpha]$ exists and varies with the rigidity of the spiral spring ❒ equation $\{d\}=[\alpha]\bullet\{F\}$ exists
	❒ the complete positioning of the structure before loading is called "*isostatic*" positioning *(a particular case of the previous one where the rigidity of the spiral spring becomes infinite).*	❒ the structure is "properly linked" ❒ the flexibility matrix $[\alpha]$ exists ❒ equation $\{d\}=[\alpha]\bullet\{F\}$ exists
	❒ the complete positioning of the structure before loading is called "*hyperstatic*" positioning *(the equations of equilibrium are no longer sufficient to obtain the link forces on the structure under loading).*	❒ the structure is "properly linked" ❒ the flexibility matrix $[\alpha]$ exists and varies with the characteristics of the links and their number ❒ equation $\{d\}=[\alpha]\bullet\{F\}$ exists

Figure 2.39. *Different connections to the surroundings for the same structure and loading*

<table>
<tr><th>“flexibility” approach of a structure</th><th>“stiffness” approach of a structure</th></tr>
<tr><td colspan="2">“n” points or nodes selected on the structure</td></tr>
<tr><td colspan="2">$\{F\}$ loading (forces, moments) acting on “n” nodes</td></tr>
<tr><td colspan="2">$\{d\}$ degrees of freedom (linear, angular displacements)
“associated” with the loading (see [2.91])</td></tr>
<tr><td colspan="2">structure not properly linked
(not sufficiently linked to its environment, i.e. in an hypostatic manner)</td></tr>
<tr><td colspan="2">behavior equation:</td></tr>
<tr><td>$\{d\} = [\alpha] \bullet \{F\}$
the flexibility matrix $[\alpha]$, inverse of the stiffness matrix $[k]$, does not exist (it cannot be defined),this equation cannot be used</td><td>$\{F\} = [k] \bullet \{d\}$
the stiffness matrix $[k]$ exists, it is a singular matrix (it cannot be inverted)</td></tr>
<tr><td colspan="2">same structure properly linked
(the previous structure is linked in an isostatic or hyperstatic manner to its surroundings</td></tr>
<tr><td colspan="2">behavior equation:</td></tr>
<tr><td>~~$\{d\} = [\alpha] \bullet \{F\}$~~
equation not usable (see above)

$\{d^*\} = [\alpha^*] \bullet \{F^*\}$
the flexibility matrix $[\alpha^*]$, inverse of the stiffness sub-matrix $[k^*]$, exist</td><td>to start with, same equation as above
$\{F\} = [k] \bullet \{d\}$
⇩
application of the linking conditions
⇩
linking actions (unknown) — displacements prevented at the links
$\begin{Bmatrix} F_\ell \\ \hdashline F^* \end{Bmatrix} = \left[\begin{array}{c:c} [k_\ell] & [k_\ell^*] \\ \hdashline [k_\ell^*]^T & [k^*] \end{array}\right] \bullet \begin{Bmatrix} d_o = 0 \\ \hdashline d^* \end{Bmatrix}$
forces applied (know) — displacements unknown
$\{F^*\} = [k^*] \bullet \{d^*\}$
the stiffness sub-matrix $[k^*]$ exists, and can be inverted</td></tr>
</table>

In the following, in order to simplify the notations, and except in special cases, the transcriptions $\{F\}$, $\{d\}$, $\{k\}$ will also be used to describe the behavior of a properly linked structure

[2.122]

(plates or shells) structures. For these kinds of structures, total integration of modeling choice in CAD software is not presently available. The designer has to intervene in the CAD model to change it into a model compatible with elements to be used[45]. For the time being, we shall only indicate the existence of other types of elements shown in the figure. These elements shall be dealt with in Chapter 5.

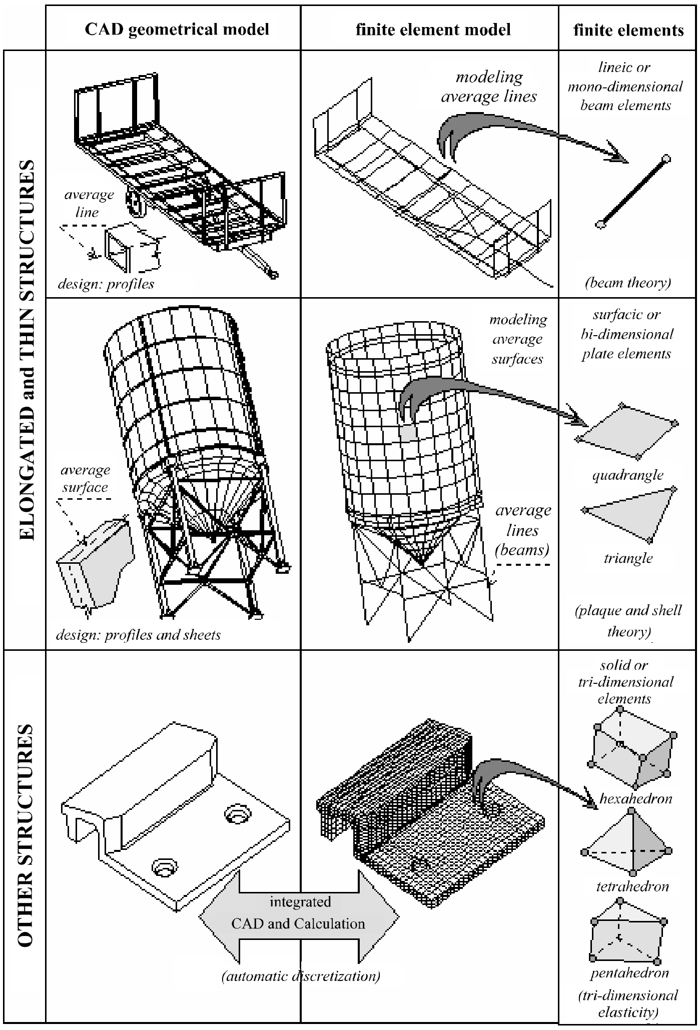


Figure 3.36. *Topology of the main types of finite elements*

45 It has already been indicated that beam elements, for example, were preferred for both precision of results and level of discretization to elements of plane stress (see section 3.2.4.5). They will also be preferred to solid elements (see Chapter 5). The same is valid for plate elements, of higher performance than solid – or volumic – elements.

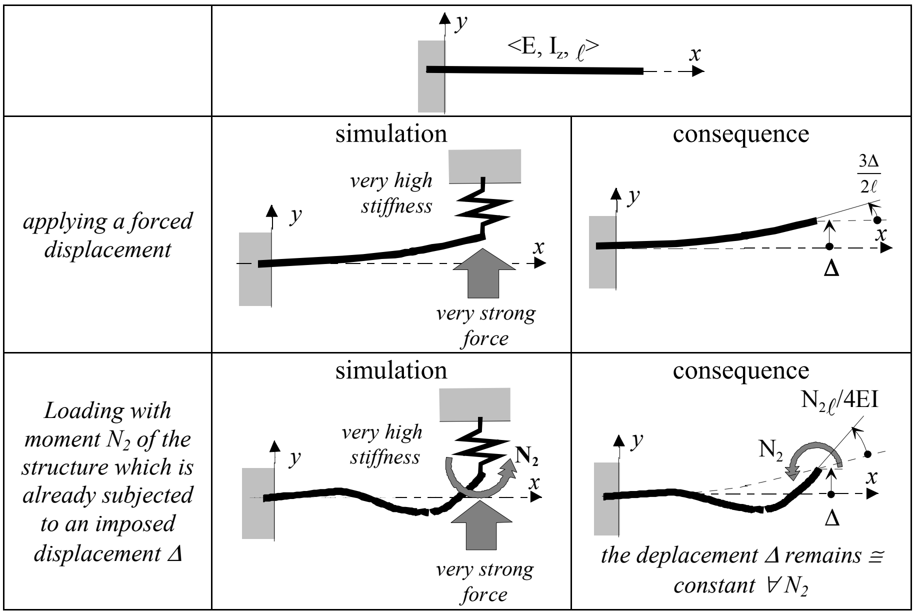


Figure 4.15. *Imposing a displacement*

NOTES

❐ This "stratagem" helps in imposing (linear or angular) displacements at specified places of the structures. These displacements remain unaffected by the intensity of the "real" loads that can be applied thereafter.

❐ This method is commonly used in calculation codes to impose non-zero boundary displacement-conditions.

4.2.4. *Assembly of a truss element and a beam element under simple plane bending*

➢ *Object:*

The finite elements for assembly are indicated in Figure 4.16.

Write the stiffness matrix of the structure obtained after assembly.

The reader can make sure that in this manner it is possible to obtain each of the terms of the global stiffness matrix as they are seen in the figure.

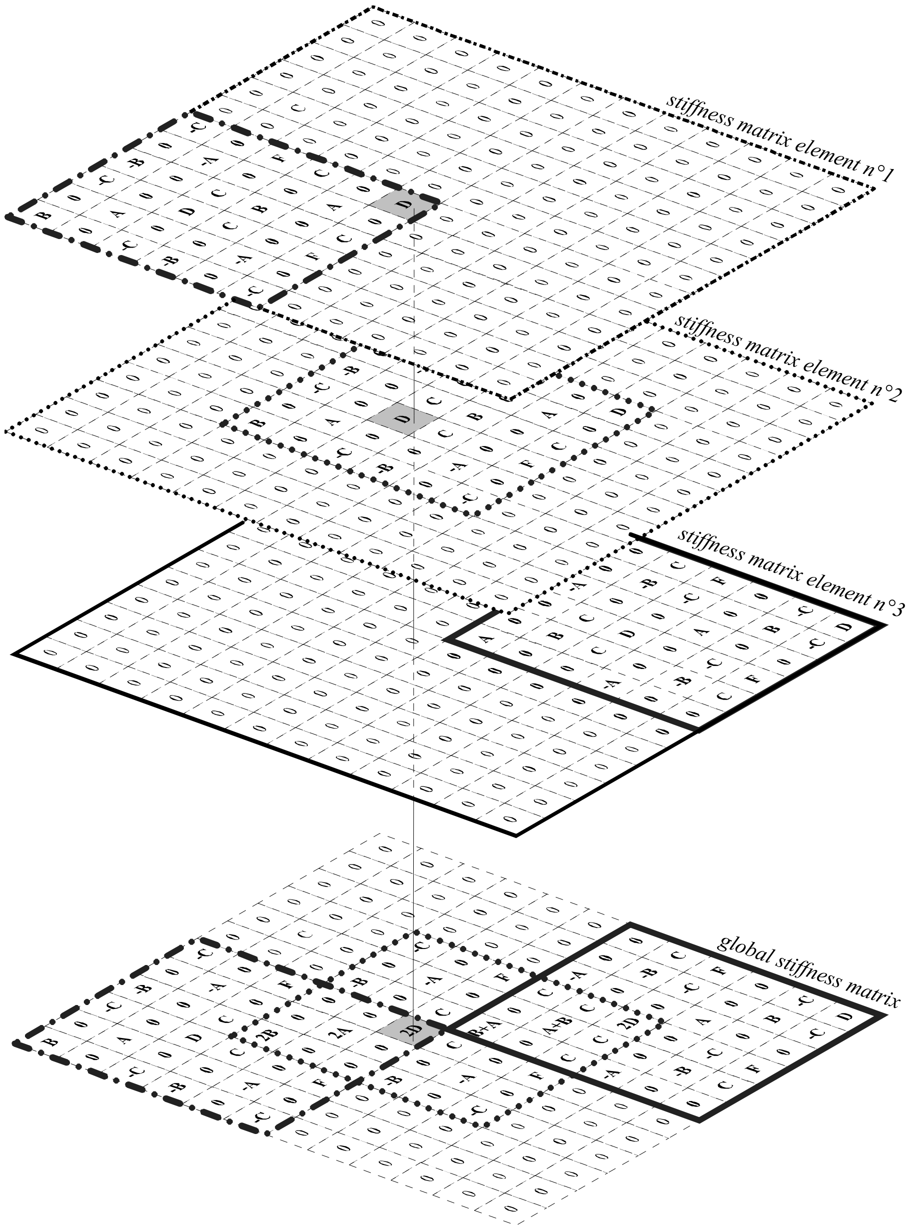


Figure 4.34. *Assembly method of the element stiffness matrices*

the behavior equation in the local system of the element. The transfer matrix defined in section 4.4.3 enables us to find the values of the dof mentioned in the local system of each element. We can then write:

$$\underset{Local}{\{F\}_{el3}} = \underset{Local}{[k]_{el3}} \bullet \underset{Local}{\{d\}_{el3}}$$

which shall give us the values of the nodal forces in the local system. These are represented on Figure 4.36. They consist of:

– internal linking actions between adjacent elements;

– external loading;

– external linking actions.

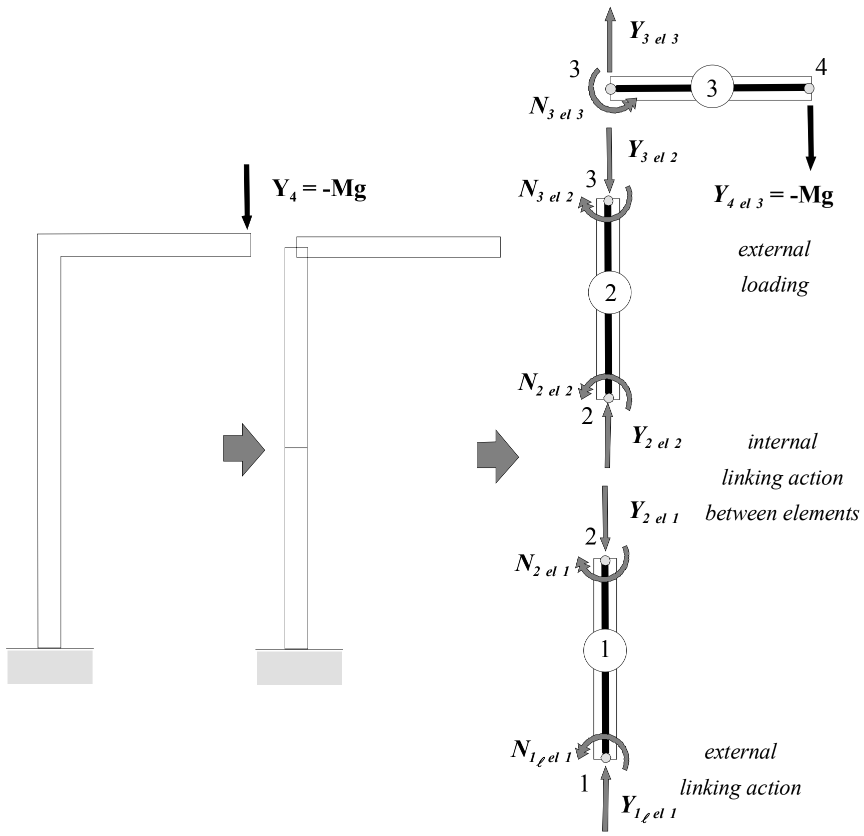


Figure 4.36. *Nodal forces (local coordinates-system): equilibrium of beam elements*

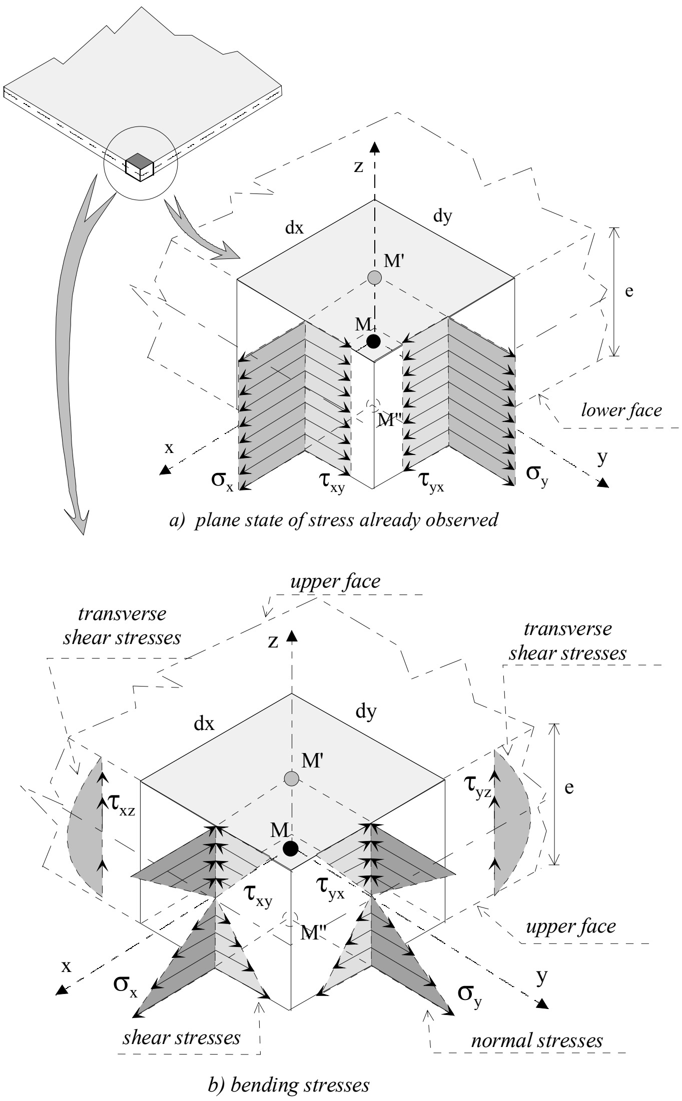


Figure 5.18. *Stresses in a bending plate*

5.4.2. *Resultant forces and moments for cohesion forces*

On the basic domain shown in Figure 5.18, we can define elementary resultant forces and moments for cohesion forces acting on the faces normal to $\vec{x}$ and $\vec{y}$. In order to do this, we use a similar approach to that in Chapter 1, section 1.5.1. The

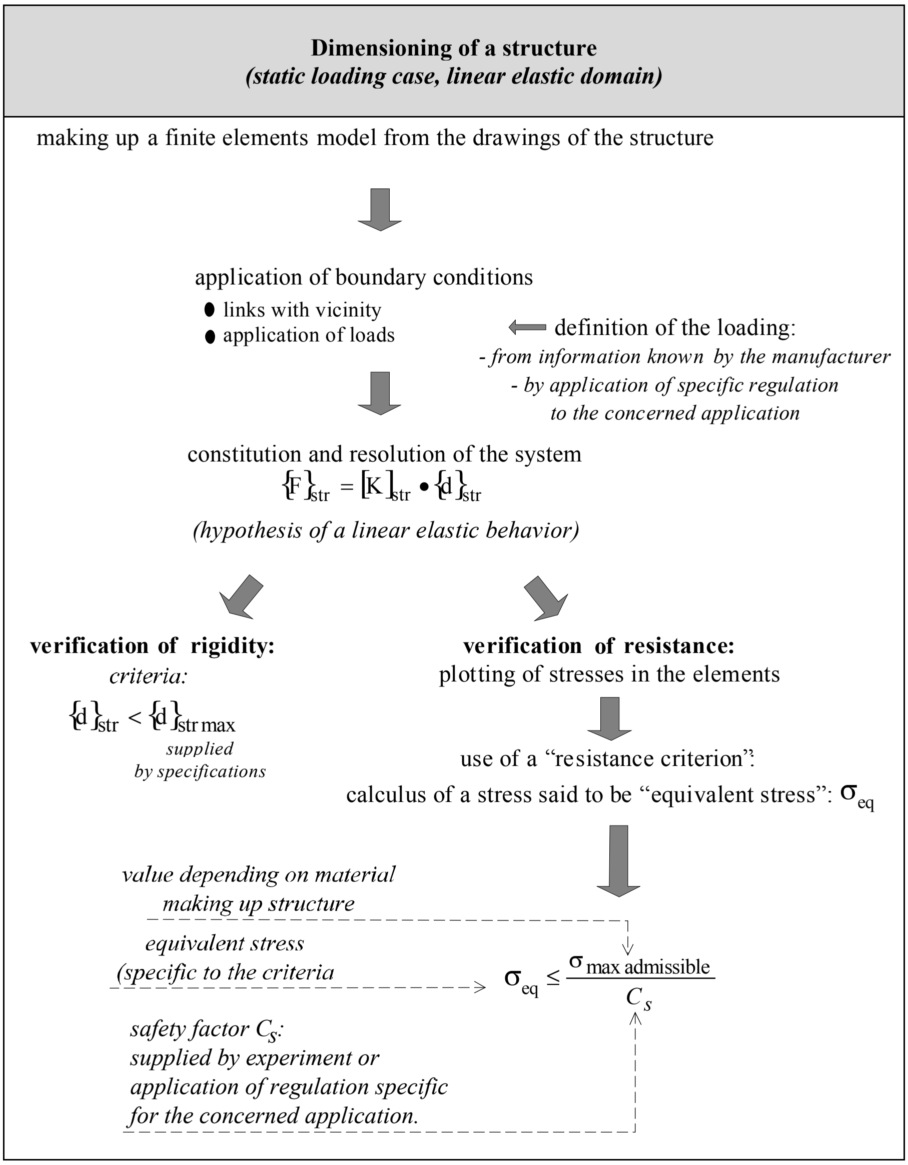


Figure 7.2. *General dimensioning procedure for static loading*

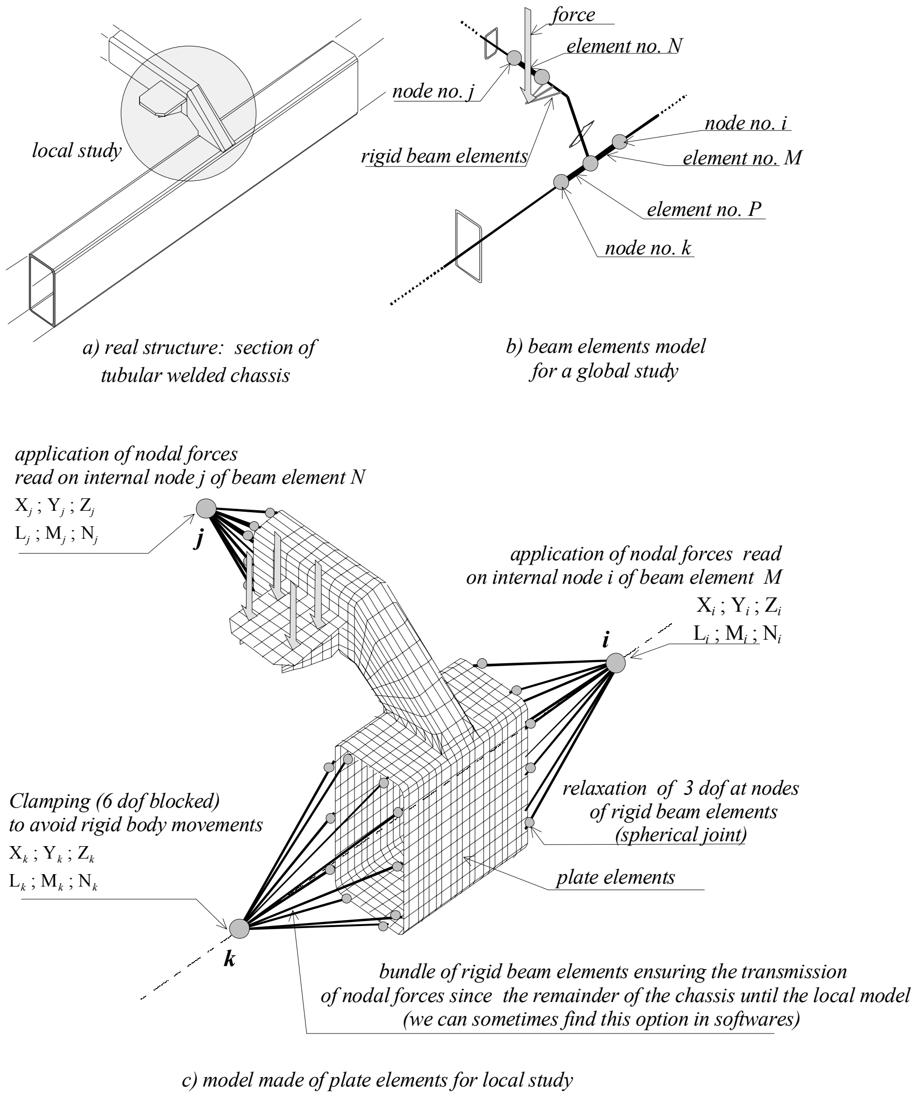


Figure 8.5. *Global model and local model*

8.51: all the translation dof T_x must be blocked. The structure then conserves a possibility of two plane rigid-body motions (middle plane (yz)). Translation T_y and rotation around axis $\vec{x}$ will be removed by canceling all the translation dof T_y of nodes on the "die-body" bearing zone.

The remaining motion of translation T_z will be eliminated by blocking any node following this direction.

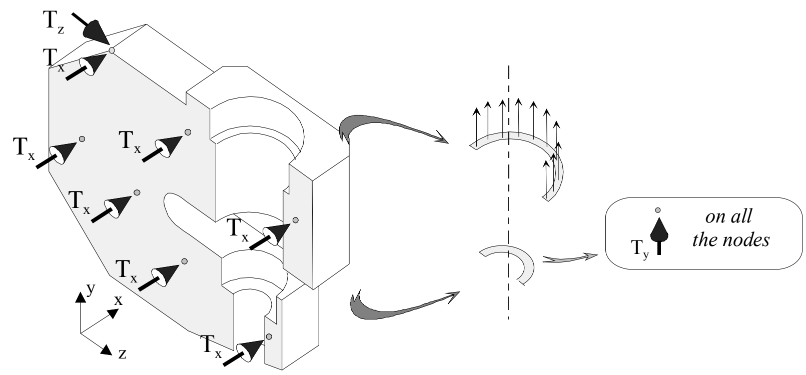


Figure 8.51. *Boundary conditions for the half body model*

8.4.2.6. *Other aspects of the modeling*

❍ Material properties

All the solid elements are in the same material (Young's modulus and Poisson's ratio).

❍ Mesh generation

The mesh generation of the body model in solid elements, done for example through integrated CAD-Finite element software, does not pose a problem. Given the weak radius of the curve (geometric singularity) and the significant effort values, the zone in the hollow of the goose neck will show large stress concentration. The operator will have to make the meshing more dense in this zone.

We can have an idea of the increased torsion flexibility of the split tube by analyzing the deflected shapes due to torsion in Figure 9.36.

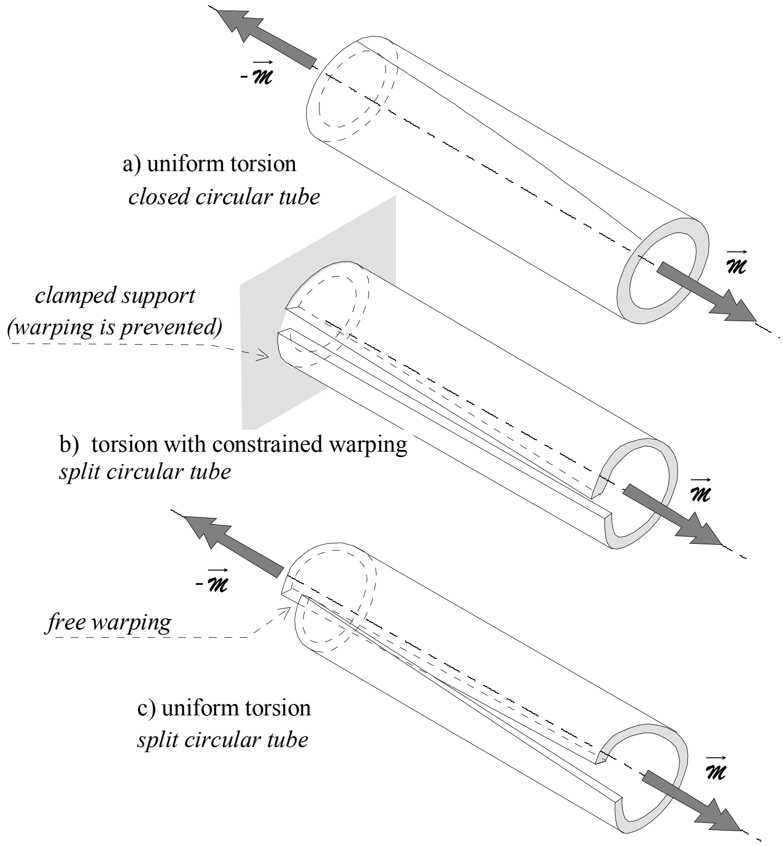


Figure 9.36. *Deflected shapes due to torsion. The torsional rigidities decrease from a) to c)*

9.3.2.6. *Torsion with constrained warping*

Figure 9.36b shows the nature of the deflected shape for the split tube whose one end is clamped. The clamped section cannot warp. It can then be easily conceived that, for an identical torsional moment, rotation due to torsion in Figure 9.36 shall be less in case b) than in case c)[35]. The torsional rigidity increases when the relative

35 As a general note, when we eliminate the displacement possibilities of certain zones of a linear elastic structure, it is evidently less deformable, i.e. more rigid.

Dimensioning of a pre-tightened bolt

estimation of the working forces on a fastener: using procedure[11.6] leads to:

– a working tensile force: X

– a working tangential force: $F_T = \sqrt{Y^2 + Z^2}$

initial tensile force necessary in the screw shank (see [11.10])

$$X_0 \geq 0.8\,X + 1.25\,\frac{F_T}{f}$$

(f: coefficient of mutual friction of the surfaces in contact)

tightening torque necessary to create this tension (approximate value)

$L_0 \cong 0.2 \times X_0 \times d$ *(d nominal diameter of the shank)*

initial stresses in the screw shank when tightening

◆*normal stress:*

$$\sigma = \frac{X_0}{s_0} \quad (s_0 = \pi\, r_0^2 \text{ minor thread root cross-section})$$

◆*maximum shear stress (approximate value):*

$$\tau = \frac{L_0}{2} \times \frac{r_0}{i_0} \quad \left(i_0 = \frac{\pi\, r_0^4}{2}\right)$$

◆*equivalent normal stress of Von Mises:*

$$\sigma_{eq.V.Mises} = \sqrt{\sigma^2 + 3\tau^2}$$

criteria of resistance

we must verify:

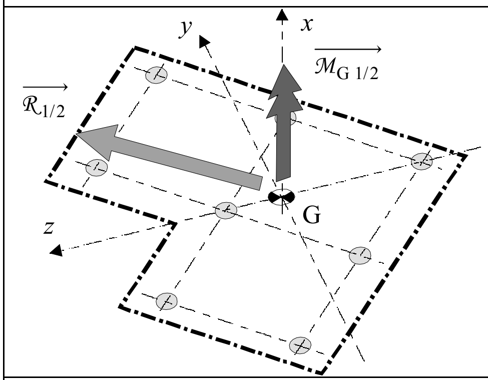
$$\Rightarrow \sigma \leq 0.7\,R_r$$

$$\Rightarrow \sigma_{eq.V.Mises} \leq R_e$$

[11.22]

11.3.5. *Summary*

The behavior of the riveted assembly may be summarized as follows.

Riveted joint fastening together parts 1 and 2
forces on "n" fasteners of center (i), each one with shank section "s"
G geometric center of "n" sections: $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} z_i = 0$; *polar quadratic moment:* $I_0 = s \times \sum_{i=1}^{n} (y_i + z_i)^2$
 *resultant force and moment of transmittable forces (already known; this is data)* $\left\{\begin{array}{l} \overrightarrow{\mathcal{R}_{1/2}} = \mathcal{T}_y \vec{y} + \mathcal{T}_z \vec{z} \\ \overrightarrow{\mathcal{M}_{G1/2}} = \mathcal{M}_x \vec{x} \end{array}\right\}_G$
shear stress on a section of center i: *components:* $\tau_{xy_i} = \frac{\mathcal{T}_y}{n \times s} - \frac{\mathcal{M}_x}{I_0} \times z_i$; $\tau_{xzi} = \frac{\mathcal{T}_z}{n \times s} + \frac{\mathcal{M}_x}{I_0} \times y_i$ *modulus:* $\tau_i = \sqrt{\tau_{xy_i}{}^2 + \tau_{xz_i}{}^2}$
corresponding shear force on section i: *components*: $Y_i = \frac{\mathcal{T}_y}{n} - \mathcal{M}_x \times \frac{z_i}{\sum_{i=1}^{n} \left(y_i{}^2 + z_i{}^2\right)}$; $Z_i = \frac{\mathcal{T}_z}{n} + \mathcal{M}_x \times \frac{y_i}{\sum_{i=1}^{n} \left(y_i{}^2 + z_i{}^2\right)}$ *modulus:* $F_{T\,i} = \sqrt{Y_i{}^2 + Z_i{}^2}$
criterion of resistance
$\tau_i \leq \frac{R_{rg}}{C_s}$ *C_s: safety factor*

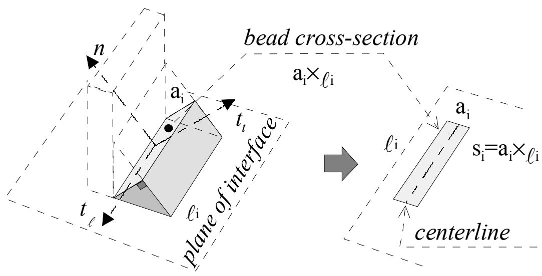
[11.23]

❒ For all practical purposes, we can rapidly characterize an “equivalent” weld section by means of specific additional software working from input of the geometry of the section, and giving the characteristics of a beam cross-section. Such software is often integrated as a specific function in a finite element software.

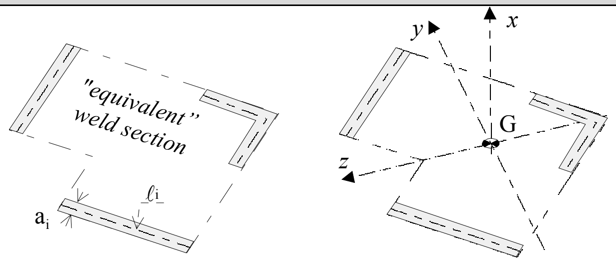
11.4.3. *Summary*

Dimensioning of a welded joint

a weld bead is reduced to its bead cross-section that is flattened against the plane of interface:



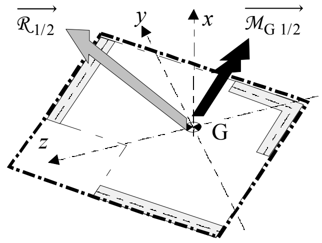
a welded joint (between parts 1 and 2) is reduced to an “equivalent” weld section



- G is the geometric center: $\int_S y\,dS = 0$; $\int_S z\,dS = 0$

- $\vec{y}$ and $\vec{z}$ are the principal quadratic axes:

$$\int_S yz\,dS = 0 \quad ; \quad I_y = \int_S z^2 dS \quad ; \quad I_z = \int_S y^2 dS \; ; \; I_0 = I_y + I_z$$



resultant force and moment of the transmittable forces (already known as data)

$$\left\{\begin{aligned}\overrightarrow{\mathcal{R}_{1/2}} &= \mathcal{N}\,\vec{x}+\mathcal{T}_y\,\vec{y}+\mathcal{T}_z\,\vec{z}\\ \overrightarrow{\mathcal{M}_{G1/2}} &= \mathcal{M}_x\,\vec{x}+\mathcal{M}f_y\,\vec{y}+\mathcal{M}f_z\,\vec{z}\end{aligned}\right\}_G$$

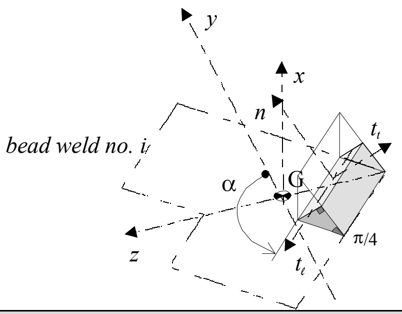
stresses on the centerlines of the "equivalent" weld section

$$\sigma_x = \frac{\mathcal{N}}{S}+\frac{\mathcal{M}f_y}{I_y}\times z-\frac{\mathcal{M}f_z}{I_z}\times y$$

$$\tau_{xy} = \frac{\mathcal{T}_y}{S}-\frac{\mathcal{M}_x}{I_0}\times z$$

$$\tau_{xz} = \frac{\mathcal{T}_z}{S}+\frac{\mathcal{M}_x}{I_0}\times y$$

regulation stresses in a bead



$$\begin{Bmatrix}\sigma_n\\ \tau_\ell\\ \tau_t\end{Bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & s & -c\\ 0 & c\sqrt{2} & s\sqrt{2}\\ 1 & -s & c\end{bmatrix}\bullet\begin{Bmatrix}\sigma_x\\ \tau_{xy}\\ \tau_{xz}\end{Bmatrix}$$

with $\alpha = \left(\vec{t_\ell},\vec{y}\right)$, $s = \sin\alpha$; $c = \cos\alpha$

resistance criterion for the bead

$$\sqrt{\sigma_n^{\,2}+3\left(\tau_\ell^{\,2}+\tau_t^{\,2}\right)} \le \frac{R_{r\ \text{bead}}}{C_s} \quad \text{with } 1 \le C_s \le 1.2$$

[11.28]

Appendix C

List of Summaries

Part I

Chapter 1. The Basics of Linear Elastic Behavior

Chapter 2. Mechanical Behavior of Structures: An Energy Approach

Chapter 3. Discretization of a Structure into Finite Elements

Part II

Chapter 5. Other Types of Finite Elements

Chapter 6. Introduction to Finite Elements for Structural Dynamics

Chapter 7. Criteria for Dimensioning

Chapter 8. Practical Aspects of Finite Element Modeling

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Complete plate elements (membrane + bending) Figure 8.2
Solid 3D elements Figure 8.3
The same structure modeled by each of the three element types Figure 8.4
Use of the software; section 8.5.3

Part III

Chapter 9. Behavior of Straight Beams

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Traction-compression [9.15]
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Chapter 10. Additional Elements of Elasticity

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Complete state of stresses; "deformations-stresses" behavior relation [10.21]
Any complete state of stresses [10.27]

Chapter 11. Structural Joints

Bolted joint of two parts 1 and 2; estimation of forces on "n" fasteners with center (i) and section "s" [11.6]
Dimensioning of a pre-tightened bolt [11.22]
Dimensioning of a riveted joint [11.23]
Dimensioning of a welded joint [11.28]

Appendices

A: Modeling of Common Mechanical Joints
B: Mechanical Properties of Materials