Chapter 6

Modeling of Thin and Line Regions

6.1. Introduction

Some devices such as the tank and accessories of a transformer, ship hulls, air-gaps in machines, contactors or magnetic recording heads, shielding, etc. are mainly made up of sheet or line type parts of thin air-gaps or cracks. Modeling these parts using traditional finite volume elements used in 3D software is tiresome, and even impossible. Moreover, the skin effect in ferromagnetic materials increases the difficulties of meshing eddy current problems in under sinusoidal conditions. To cope with these difficulties, it is possible to use special "shell elements" for the modeling of magnetic or thin conducting regions, "surface impedance" elements for the modeling of conducting regions having a low skin depth, etc. This chapter presents these special elements.

6.2. Different special elements and their interest

We call a "thin region", an region which has a low thickness compared to its other dimensions, and "neighbor regions", the external regions which have a common border with the thin region.

Electrotechnical devices are also made of line type parts such as clamping beams, bus bars of transformers, windings, etc.

Chapter written by Christophe GUÉRIN.

Meshing thin regions and line regions using an automatic mesh generator, which generates tetrahedral elements in 3D (and triangular elements in 2D), result, because of the low thicknesses or weak sections, in a very significant number of elements or in elements which are too long. Some problems are difficult, even impossible, with current computing tools. The tetrahedral or triangular finite elements can have low accuracy when they are too long.

An alternative to this difficulty of meshing the thin regions is the use of a mapped mesh generator or by extrusion which generates hexahedral or prismatic elements in 3D (quadrangular in 2D). The latter withstands a strong lengthening, which is not easy and takes time. The other way based on the use of an automatic mesh generator, consists of using special elements which allow the thin regions to be modeled by surfaces and line type regions to be modeled by lines. Thus, the description of the geometry and the mesh is largely simplified. The physical phenomena which occur inside these regions are taken into account in the integral formulation by surface or linear terms. The average surface (called Γ thereafter) which will describe a thin region will pass through the middle of this region.

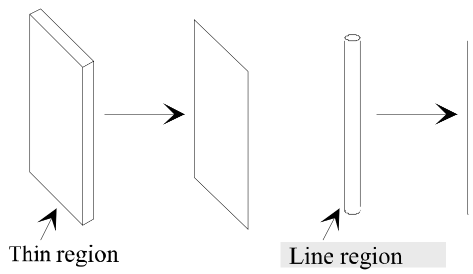


Figure 6.1. *Modeling of a thin region and a line type region with special elements*

For some magnetoharmonic problems, the solid conductors are characterized by a strong skin effect. When the skin depth is small compared to the characteristic dimension of the conductor with a material with linear properties, the physical quantities such as the current or the magnetic field have a known exponential decay. The meshing of the conducting region with traditional volume elements must consist of elements which are smaller than the size of the skin depth. This situation will lead, for some problems, to a very high number of elements. Special surface elements, using the concept of surface impedance, which describe the surface of the conducting region, allow the exponential decay to be taken into account. They also allow the magnetic field to only be calculated on the surface and outside. The problems which are characterized with a low skin depth and which can require the use of such elements are, for example, problems of induction heating and problems relating to the transformers (tanks). Let us now consider a magnetoharmonic problem where there are thin regions of low thickness in which eddy currents flow. When the skin depth is lower than the thickness of the thin region and its material is linear, the variation of the quantities along the thickness is exponential.

The common characteristic of special elements is to suppose known the variation of the physical quantities along the thickness of the thin region or the skin depth. We call "shell elements" the surface elements of a thin region to be described (see Figure 6.2).

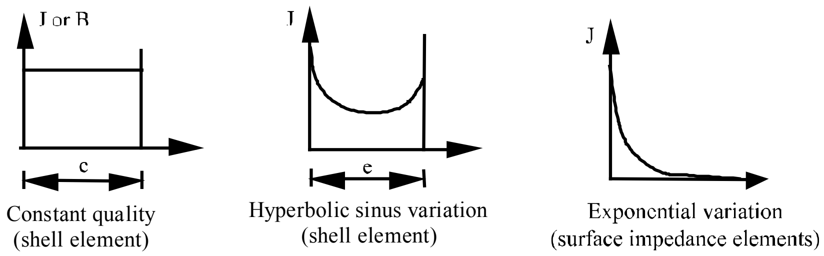


Figure 6.2. *Various types of special surface elements ranked according to the variation of the physical quantities along the thickness*

We can also rank the special elements according to types of unknown variables used by the associated formulations (fields, vector potentials, scalar, electric, magnetic). According to the type of physical problem that represents a thin region, and according to the unknown variable type, we have two types of shell elements: elements "without potential jump" and elements "with potential jump". If the potential is constant through the thickness, the element is known as "without potential jump", otherwise, it is known as "with potential jump" and the unknown variables are duplicated in each node of the element (see Figure 6.3).

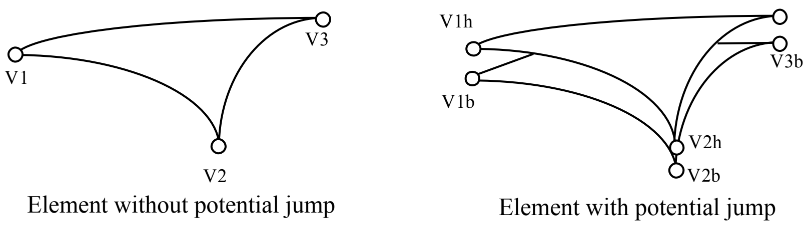


Figure 6.3. *Element without potential jump and element with potential jump*

Let us take the example of a thin plate consisting of a ferromagnetic material of high permeability. The plate is surrounded by air. The induction and the field in this plate are mainly tangent since the flux is channeled by it. If the total scalar potential is used, the magnetic field is written H = –grad ϕ, the equipotential surfaces are perpendicular to the surface of the plate. Thus, the potential at point A on a side of the plate will be equal to the potential at point B which is opposite (see Figure 6.4).

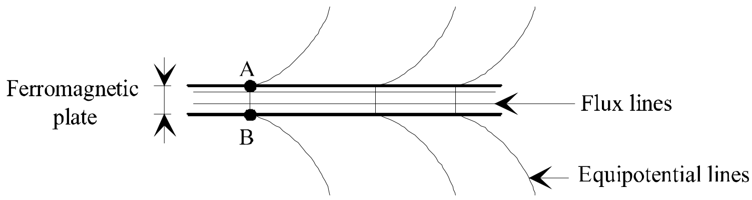


Figure 6.4. *Constant potential ϕ through the thin region*

The modeling of the plate will require only one special element without a potential jump [BRU 91]. Let us note that in scalar potential, the continuity of the tangential component of the magnetic field is exactly assured by the nodal finite elements. It can be shown that the use of the magnetic vector potential requires an element with a potential jump for the modeling of the plate. On the other hand, for the dual problem, the thin air-gap of a magnetic circuit, where the induction is mainly normal, a special element with vector potential could be without jump and an element in scalar potential will necessarily be with potential jump [NAK 90]. The nodal finite elements exactly ensure the normal component of the induction in vector potential. We can extrapolate these observations for the other magnetostatic, magnetoharmonic, transient magnetic formulations, etc. which comprise various types of thin regions. They are, for example, a magnetic circuit air-gap, a ferromagnetic plate in the air, a conducting plate in the air, a crack of a low thickness slightly conducting in a conductor, etc. The following table indicates, for the potentials used in the thin regions and the neighboring regions, if the special element requires a potential jump or not.

The use of special elements for the modeling of thin regions has other advantages:

– thickness e of the thin region which can be changed without modifying the geometry or the mesh in order to carry out parametric studies easily according to this thickness e;

– the time of calculation which is reduced compared to the use of traditional volume elements [NAK 90].

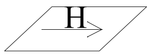
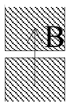
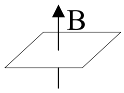
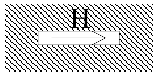
	Magnetostatics		**Magnetodynamics**	
	Ferromagnetic thin sheet	Air gap	Conducting thin sheet	Crack
				
	$\mu_e << \mu$	$\mu_e >> \mu$	$\sigma_e >> \sigma$	$\sigma_e << \sigma$
Formulations in H	ϕ, ϕ_r		Tϕ H	
ϕ, ϕ_r, Tϕ H	***Without jump***	*Jump*	*Jump*	***Without jump***
Formulations in B	A		AV, A*, E	
A, AV, A*, E	*Jump*	***Without jump***	***Without jump***	*Jump*

Table 6.1. *Element with jump and element without jump of the unknown variable. μ_e, σ_e: permeability and conductivity of the thin region, μ, σ: permeability and conductivity of the neighboring regions*

6.3. Method for taking into account thin regions without potential jump

The method presented here allows any type of thin region to be described with any nodal or edge approximation formulation by surface elements without potential jump [BRU 91]. This method is valid in the case of thin regions where the potentials are of constant thickness, i.e. in which the physical quantities, such as the magnetic field and current density, do not vary with the thickness. Let us denote by e, the thickness of the thin region. The process of obtaining the formulation for the thin region consists of decomposing the volume integrals of the volume formulation into a line integral along thickness e and a surface integral along surface $\Gamma$of the thin region.

$$\int_\Omega F d\Omega = \int_\Gamma \left(\int_e F\, dz \right) d\Gamma \qquad [6.1]$$

where F is the integrating term of the integrals of the volume region formulation.

As the potentials and the physical quantities are considered constant along the thickness, the linear integral is obvious. It is worth eF. Thus, the terms of the finite elements formulation for the thin region are obtained by transforming the terms of the formulation for volume elements into surface integrals, by multiplying them by thickness e and using the shape functions of surface elements instead of those of volume elements.

We take, as an example, the formulation for the total magnetic scalar potential. The linear system for this formulation for volume regions is written (see [BRU 91]):

$$[A][\phi]=[C] \qquad [6.2]$$

with $[A_{ij}] = -\int_\Omega grad\ w_i^t \cdot [\mu]\, grad\ w_j\ d\Omega$,

$$[C_i] = -\int_\Omega grad\ w_i^t \cdot B\ d\Omega$$

where B is the induction.

The matrix terms for the formulation of thin regions can be written:

– for $[A_{ij}]$: $-\int_\Gamma e\ grad_s\ w_i^t \cdot [\mu]\, grad_s\ w_j\ d\Gamma$;

– for $[C_i]$: $-\int_\Gamma e\ grad_s\ w_i^t \cdot B\ ds$.

6.4. Method for taking into account thin regions with potential jump

The idea is to consider a surface element with potential jump as a prismatic element. We make the assumption that the potential is considered linear in the direction of the element thickness. Thus, quantities such as the magnetic field are supposed to be constant in the thickness. The prismatic element has a first order interpolation function along the thickness and any order along the other directions. We will integrate the interpolation functions along the thickness in order to obtain the formulation of an element with potential jump [SUR 86] [POU 93]. Two valid methods for a nodal approximation are successively presented [GUE 94a]. The formulation in total scalar potential is taken as an example for the application of these methods. In the first method, the integrals of the shape functions along the thickness are calculated in an analytical way before assembly and any numerical processing. In the second method, which is a more general approach, the integration is performed numerically, at the time of integration of the matrix terms in the matrix.

We assume here that the elements with potential jump have their nodes duplicated. The duplicated nodes are at the same coordinates as those of the origin.

6.4.1. *Analytical integration method*

We consider a reference coordinate system related to the element. Let us write x,y for the tangential curvilinear coordinates on average surface Γ of the element and z for the one normal on the surface of the element. We must consider curvilinear coordinates x,y,z of the real element expressed in the local reference coordinate system and not those of the reference element, in order to take into account the thickness of the element. Thickness e is assumed to be constant in each element. The magnetic scalar potential ϕ is interpolated using the approximation functions of w'_i of the prismatic element which has n' = 2 n nodes:

$$\phi = \sum_{i=1}^{n'} w'_i(x,y,z)\phi_i \qquad [6.3]$$

Functions w'_i and w'_{i+n} differ only by the terms according to z, which allows the Lagrangian functions $w_i(x,y)$ to be defined. These functions are the surface interpolation functions on average surface Γ and $\lambda_1(z)$ and $\lambda_2(z)$ which are the Lagrange interpolation functions in the thickness, i.e., those of a nodal line element of the 1st order with 2 nodes of length e. The potential is written:

$$\phi = \sum_{i=1}^{n} w_i(x,y)\lambda_1(z)\,\phi_{i1} + \sum_{i=1}^{n} w_i(x,y)\lambda_2(z)\,\phi_{i2} \qquad [6.4]$$

with $\lambda_1 = \left(\frac{1}{2}+\frac{z}{e}\right)$, $\lambda_2 = \left(\frac{1}{2}-\frac{z}{e}\right)$ and $\phi_{i1} = \phi_i, \phi_{i2} = \phi_{i+n}$, $i \in [1,n]$

Indices "1" and "2" refer respectively to sides "1" and "2" of the element with potential jump. Thin region Ω is described by the formulation in total scalar potential. The volume terms corresponding to the thin region are written in their discrete form:

$$\sum_{j=1}^{n'}\left[\int_{\Omega} \mu \; grad\; w_i' \cdot grad\; w_j'\,\phi_j \, d\Omega\right] \qquad [6.5]$$

We are interested now in n first equations which correspond to the nodes on side "1" of the element. They are written:

$$\int_\Omega \mu \left\{ \sum_{j=1}^{n} \left\langle \frac{\partial w_i}{\partial x}\lambda_1 \quad \frac{\partial w_i}{\partial y}\lambda_1 \quad \frac{w_i}{e} \right\rangle \cdot \left[\begin{Bmatrix} \frac{\partial w_j}{\partial x}\lambda_1 \\ \frac{\partial w_j}{\partial y}\lambda_1 \\ \frac{w_j}{e} \end{Bmatrix} \phi_{j1} + \begin{Bmatrix} \frac{\partial w_j}{\partial x}\lambda_2 \\ \frac{\partial w_j}{\partial y}\lambda_2 \\ -\frac{w_j}{e} \end{Bmatrix} \phi_{j2} \right] \right\} d\Omega = 0 \qquad [6.6]$$

The other n equations which correspond to the nodes on side "2" of the element are obtained from the first n equations by permuting potentials ϕ_{j1} and ϕ_{j2}. The volume integral on volume Ω is decomposed in a surface integral on Γ and an integral along z (F is the integrating term of [6.6]):

$$\int_\Omega F d\Omega = \int_\Gamma \left(\int_{-e/2}^{e/2} F dz \right) d\Gamma \qquad [6.7]$$

After integration and passage into the non-discretized integral form along z, the total formulation is thus written, using the surface gradient operators:

$$\begin{aligned} &\int_{\Omega_1} \mu_1 grad w_i \cdot grad \phi_1 d\Omega_1 + \int_\Gamma \frac{e\mu}{3} grad_s w_i \cdot grad_s \phi_1 d\Gamma \\ &+ \int_\Gamma \frac{e\mu}{6} grad_s w_i \cdot grad_s \phi_2 d\Gamma + \int_\Gamma \frac{\mu}{e} w_i \phi_1 d\Gamma - \int_\Gamma \frac{\mu}{e} w_i \phi_2 d\Gamma = 0 \end{aligned} \qquad [6.8]$$

$$\begin{aligned} &\int_{\Omega_2} \mu_2 grad w_i \cdot grad \phi_2 d\Omega_2 + \int_\Gamma \frac{e\mu}{3} grad_s w_i \cdot grad_s \phi_2 d\Gamma \\ &+ \int_\Gamma \frac{e\mu}{6} grad_s w_i \cdot grad_s \phi_1 d\Gamma + \int_\Gamma \frac{\mu}{e} w_i \phi_2 d\Gamma - \int_\Gamma \frac{\mu}{e} w_i \phi_1 d\Gamma = 0 \end{aligned} \qquad [6.9]$$

The system of equations above is symmetric. The second equation can be deduced from the first by permuting indices "1" and "2".

6.4.2. *Numerical integration method*

Unlike the previous method where an analytical integration was carried out, the integration along the surface and along the thickness is performed numerically, using the Gauss method, at the moment of integration of the matrix terms into the matrix, as for a volume element. We must consider the local coordinates u,v,w on the reference element. The approximation functions of the surface element with potential jump are calculated as for a prismatic element: the product of the approximation functions of a surface element without potential jump along Γ with those of a 2-node line element along the thickness is produced:

$$w_i\,(u,v,w) = w_{sj}\,(u,v)\,.\,w_{\ell k}\,(w)\,,\ i \in [1\,,\,n],\ j \in [1\,,\,n_s],\ k \in [1\,,\,2]$$

where w_{sj} (u,v) (with $j \in [1\,,\,n_s]$) are the shape functions of the surface element, $w_{\ell k}$ (w) (with $k \in [1\,,\,2]$) are those of the line element, n_s is the number of nodes of the surface element and $n = 2\,n_s$ is the number of nodes of the surface element with potential jump. The derivative of functions w_i with respect to the local coordinates are written:

$$\frac{\partial\, w_i}{\partial\, u} = \frac{\partial\, w_s}{\partial\, u}\, w_\ell\,, \qquad \frac{\partial\, w_i}{\partial\, v} = \frac{\partial\, w_s}{\partial\, v}\, w_\ell\,, \qquad \frac{\partial\, w_i}{\partial\, w} = w_s\,\frac{\partial\, w_\ell}{\partial\, w} \tag{6.10}$$

We must now express the Jacobian matrix [J] 3 × 3 of the transformation of the real element with potential jump to the reference element. Let $[J_1]$ be the 2×3 matrix formed by the first two lines of [J], and $[J_2]$ be the 1×3 matrix formed by the third line of [J]. Matrix $[J_1]$ is calculated as follows:

$$[J_1] = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} & \dfrac{\partial z}{\partial u} \\ \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} & \dfrac{\partial z}{\partial v} \end{bmatrix} = \sum_{i=1}^{n} \left(\begin{Bmatrix} \dfrac{\partial w_i}{\partial u} \\ \dfrac{\partial w_i}{\partial v} \end{Bmatrix} \cdot \langle x_i \quad y_i \quad z_i \rangle \right) \tag{6.11}$$

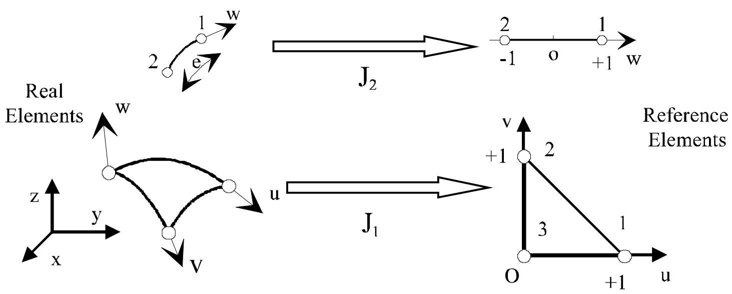


Figure 6.5. *Transformations of J_1 and J_2*

$[J_1]$ is in fact the matrix of the transformation of the real surface element without potential jump, into the reference surface element without potential jump. $[J_2]$ is the matrix of the transformation of the real line element into the reference line element (see Figure 6.5). Let n_u and n_v be two orthonorms, at coordinates (u, v) in reference coordinate system (O,u,v) of the reference element. Vector n_u, respectively n_v, is parallel to vector Ou, respectively Ov. In reference coordinate system (O,x,y,z) of the real element, they are two tangential orthogonal vectors on the surface of the surface element point (x (u,v), y (u,v), z (u,v)). Matrix $[J_1]$ is formed of the two transposed vectors n_u and n_v. Let n_w be the normal unit vector on the surface. Matrix $[J_2]$ is formed by the transposed vector n_w multiplied by the half thickness e of the thin region, as indicated below:

$$[J_2] = \begin{bmatrix} \frac{\partial x}{\partial w} & \frac{\partial y}{\partial w} & \frac{\partial z}{\partial w} \end{bmatrix} = \frac{e}{2} \cdot n_w^T \qquad [6.12]$$

In [6.12] n_w^T is multiplied by e/2, as the length of the line element is worth e in the real reference coordinate system (O, x, y, z) and is worth 2 in the reference coordinate system of the reference element (O, u, v, w). The polynomial derivatives with respect to the global coordinates (x, y, z) are given by:

$$grad w_i = \begin{Bmatrix} \frac{\partial w_i}{\partial x} \\ \frac{\partial w_i}{\partial y} \\ \frac{\partial w_i}{\partial z} \end{Bmatrix} = \left[J^T\right]^{-1} \begin{Bmatrix} \frac{\partial w_i}{\partial u} \\ \frac{\partial w_i}{\partial v} \\ \frac{\partial w_i}{\partial w} \end{Bmatrix} \qquad [6.13]$$

The general term Aij of the linear system matrix corresponding to the formulation in total scalar potential is written like an integral on the real element:

$$\iint_{e_s} \int_{e_l} \left[\sum_{j=1}^{n} \left(\mu \, grad w_i^T \cdot grad w_j \right) \phi_j \right] de_s \, de_l \qquad [6.14]$$

After passage of the global coordinates to the local coordinates, this term becomes:

$$\int_{u=-1}^{u=1} \int_{v=-1}^{v=1} \int_{w=-1}^{w=1} \left[\sum_{j=1}^{n} \left(\mu \, grad w_i^T \cdot grad w_j \right) \phi_j \right] \det[J] \, du\,dv\,dw \qquad [6.15]$$

The integration on the element is carried out using the Gaussian-quadrature method. The functions which are integrated along the thickness are second-order polynomials of the form $\left(\frac{1}{2} \pm \frac{z}{e}\right)\left(\frac{1}{2} \pm \frac{z}{e}\right)$. Two Gauss points lead to an exact integration of these functions, at least except for the numerical errors, the Gaussian-quadrature method integrating exactly a polynomial of order 2m-1 with m points of integration.

The method presented here is general and easy to implement. In fact, it applies to any formulation: the integration and the assembly of the surface element with potential jump are performed in the software in a similar way to the integration and the assembly of a traditional volume element. The difference in treatment between the two types of elements lies only in the retrieval of the interpolation functions and derivatives of these functions with respect to the coordinates: for a surface element with potential jump, the functions of the line element and those of the surface element are combined.

6.5. Method for taking thin regions into account

The method presented here is similar to the method described in section 6.3 for taking into account thin regions without potential jump. It makes it possible to describe any type of line region with any formulation with nodal or edge interpolation by line elements. This method is valid in the case of line regions where the potentials are constant in the section, i.e. in which the physical quantities, such as the magnetic field and current density, do not vary in the section. The method is deduced from the one described in section 6.3, by considering section s of the line region instead of thickness e of the thin region [BRU 91]. The formulation for the line region consists of breaking up the volume integrals of the volume formulation into a surface integral along section s and a linear integral along line λ of the line region.

$$\int_{\Omega} F \; d\Omega = \int_{\lambda} \left(\int_{s} F \; dz \right) d\Gamma \qquad [6.16]$$

where F is the integrating term of the integrals of the formulation for volume regions. As the potentials and the physical quantities are considered constant along the thickness, the surface integral is obvious. It has a value sF. Thus, the terms of the finite element formulation for the line region are obtained by transforming the terms of the formulation for volume elements into line integrals, by multiplying them by section s and by using the shape functions of the line elements instead of those for volume elements. For example, for the formulation in total magnetic scalar potential of section 6.3, the matrix terms for the formulation for line regions are written:

– for $\left[A_{ij}\right]$: $\quad -\int_{\lambda} s\ grad_{\ell} w_i^t \cdot [\mu] grad_{\ell}\ w_j\ d\lambda$;

– for $\left[C_i\right]$: $\quad -\int_{\lambda} s\ grad_{\ell}\ w_i \cdot \vec{B}\ d\lambda$.

6.6. Thin and line regions in magnetostatics

6.6.1. *Thin and line regions in magnetic scalar potential formulations*

6.6.1.1. *Thin and line regions without potential jump*

Since the surface or line elements are without potential jump, we take into account only the surface gradients in thin or line regions without potential jump in magnetic scalar potential formulations. The magnetic scalar potentials make it possible to take into account the regions in which fields H and B are mainly tangent with the thin or line region. The permeable regions and regions with tangent magnetizations can then be taken into account [BRU 91]. The surface and line formulations without potential jump are obtained from the volume formulation thanks to the methods described in sections 6.3 and 6.5.

6.6.1.2. *Thin regions with potential jump*

In thin regions with potential jump in scalar potential formulations, fields H and B can have any direction [GUE 94a]. Fields H and B must be constant in the thickness of the region. Such regions thus make it possible to describe thin air-gaps, permeable regions, as well as regions with magnetization of any direction. The surface formulation with potential jump is obtained from the volume formulation thanks to the method described in section 6.4.

6.6.1.3. *Air-gap edges in magnetic scalar potentials*

When a magnetic circuit with a thin air-gap is described with magnetic scalar potentials, there is a potential jump on all the surface of the air-gap, which is prolonged in the air, beyond the edge. When this air-gap is described by a thin region with potential jump, this thin region must be prolonged in the air, until the potential jump is assumed to be negligible. On the edge of the thin region of the prolonged air-gap, the degrees of freedom on the two sides are then confounded ($\phi_1 = \phi_2$) [GUE 94a].

6.6.1.4. *Example: magnetic circuit with air-gap*

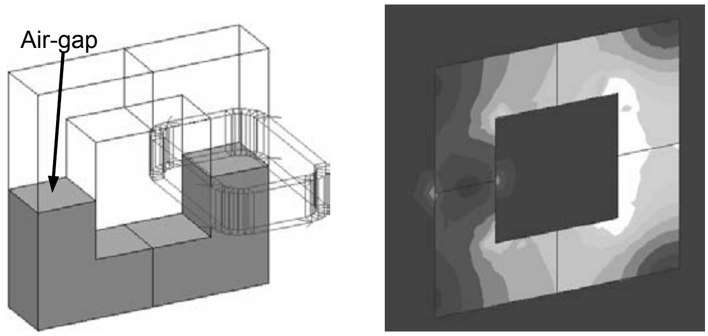


Figure 6.6. *Magnetic circuit with thin air-gap in magnetic scalar potential (reduced in the air and the air-gap, total in the magnetic circuit). On the right: isovalues of induction on the vertical symmetry plane*

6.6.2. *Thin and line regions in magnetic vector potential formulations*

6.6.2.1. *Thin and line regions without potential jump*

Thin regions without potential jump in a magnetic vector potential formulation make it possible to take into account the surface currents (layers) and the thin air-gaps, and the line regions to take into account the linear currents [NAK 90], [BRU 91]. The surface and line formulations without potential jump are obtained from the volume formulation thanks to the methods described in sections 6.3 and 6.5.

6.6.2.2. *Thin and line regions with potential jump*

In thin regions with potential jump in nodal vector potential formulation, fields H and B can have any direction. Fields H and B must be constant in the thickness of the region. Such regions then make it possible to describe thin magnetic regions, thin air-gaps or surface currents (layers) [GUE 94a]. The surface formulation with potential jump is obtained from the volume formulation thanks to the method described in section 6.4.

6.7. Thin and line regions in magnetoharmonics

In magnetoharmonics, for eddy current problems, special elements are used in the following cases:

– solid conducting regions where the skin effect is strong: the skin depth is much lower than characteristic dimension of the thin region;

– conducting thin regions. Several cases can arise depending on whether the skin depth is higher or lower than the thickness of the thin region;

– conducting line regions;

– slightly conducting or insulating thin regions in a solid conducting region.

6.7.1. *Solid conducting regions presenting a strong skin effect*

6.7.1.1. *The surface impedance condition*

For a linear, homogenous and isotropic material, the skin depth in the conductors is calculated by:

$$\delta = \sqrt{2/(\sigma\omega\mu)} \qquad [6.17]$$

Meshing difficulties appear when skin depth δ becomes smaller compared to the characteristic dimension of the solid conductors to be modeled. This situation occurs when either the frequency, permeability or resistivity is high. Surface impedance Z_S connects the tangential component to the surface of the conductor of the magnetic field H to the tangential component of the electric field E by the following relation:

$$n \times E = Z_S n \times (n \times H) \qquad [6.18]$$

where n is the unit vector normal at the surface and outgoing from the conducting region. In order to obtain a first expression of the surface impedance, we must consider the problem of a plate with an infinite thickness subjected to a uniform sinusoidal field parallel with the side of the plate which is composed of a linear material. This one-dimensional problem is solved in [STO 74]. The complex impedance found is constant, i.e. independent of the value of the field. It will be called "surface impedance in the one-dimensional (or 1D) approximation":

$$Z_S = \frac{|H_S|}{|E_S|} = \frac{1+j}{\sigma\delta} \qquad [6.19]$$

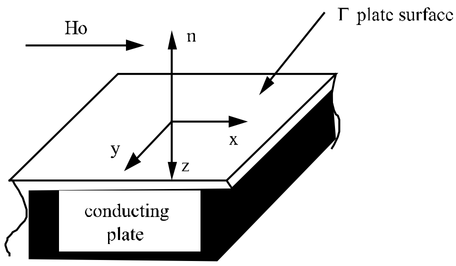


Figure 6.7. *Plate of infinite thickness in a uniform field*

This surface impedance associated with the finite element method is used on any geometries and not only plane ([KRA 88], for example). In 2D, a formulation in vector potential using the 1D approximation can be used [HOO 85]. In 3D, the magnetic scalar potential is generally used as state variable [GUE 94a], sometimes the magnetic vector potential A associated with the electric scalar potential V [LOU 95]. For the description of the regions outside the conductors, the method of boundary integrals can be used [KRA 88] [TAN 88]. In this case, only the surface of the conductor has to be meshed, but the rigidity matrix is full. The finite element method can also be used, which leads to a sparse band matrix [ROD 91], [GUE 94a].

6.7.1.2. *Validity and limitations of the surface impedance condition*

There is a limitation of topological order, which is related to the magnetic scalar potential. In fact, when the conducting region is non-simply connected, i.e. it comprises at least one hole, this potential cannot be used without specific processing. There are also limitations of a geometric nature, during the use of the expression of the surface impedance in the one-dimensional approximation. This expression is valid if the following conditions are checked:

– $\delta \ll L$ L: characteristic length of the conductor;

– $\delta \ll R$ R: characteristic curvature radius of the conductor.

Moreover, the one-dimensional approximation is no longer valid when the problem consists of edges or corners, or when the radius of curvature is small compared to the skin depth. Surface impedance formulae modified for small curvature radii, for edges with 90° or for an edge of any θ angle are proposed in [DEE 90] [JIN 93].

Strong features	Weak features
– Easy implementation – Low CPU time cost – Good accuracy when the skin depth is small – No volume meshing inside the conductors	– Bad accuracy in the corners and edges if the one-dimensional value of the surface impedance is used – Taking into account conductors which are non-simply connected is impossible without specific processing

Table 6.2. *Strong and weak features of the surface impedance method in magnetic scalar potential compared to a volume formulation H, E, AV, T-Ω*

6.7.1.3. *The surface impedance condition in magnetic scalar potential*

Several presentations of the surface impedance formulation in reduced scalar potential exist: [BOS 84], [TAN 88] or [KRA 88]. The formulation presented in this section is taken from [GUE 94a]. The materials must have an isotropic permeability and an isotropic linear conductivity.

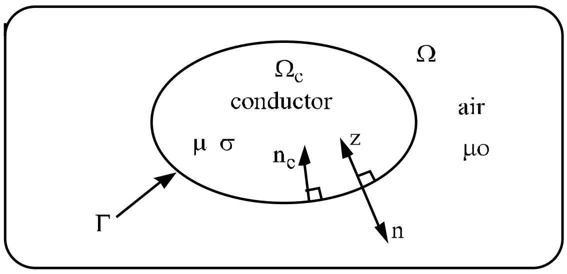


Figure 6.8. *Notations of the formulation*

The quantities subscripted by “0” are the values of these quantities on surface Γ of conductor Ω_c. Those subscripted by “s” are the quantities on Γ tangential to Γ. By considering the case of the plate of infinite thickness [STO 94], the variation of the quantities along direction z is exponential:

$$E(z) = E_0 e^{-(1+j)\frac{z}{\delta}}, \quad H(z) = H_0 e^{-(1+j)\frac{z}{\delta}}, \quad J(z) = J_0 e^{-(1+j)\frac{z}{\delta}} \qquad [6.20]$$

The tangential magnetic field is expressed with tangential source field H_{js} and reduced scalar potential ϕ_r:

$$H_s = H_{js} - grad_s\ \phi_r \qquad [6.21]$$

The formulation in reduced potential, in the air region Ω, is recalled below:

$$\int_\Omega \mu_0 \operatorname{grad} w \cdot \operatorname{grad} \phi_r\, d\Omega + \int_\Gamma wB \cdot \mathrm{n_e}\, d\Gamma = \int_\Gamma \mu_0 w H_j \cdot \mathrm{n_e}\, \mathrm{d}\Gamma \qquad [6.22]$$

where n_e is the outgoing normal of region Ω: $n_e = -n$.

The second term of [6.22] is transformed thanks to surface impedance relation [6.18] in order to take into account the conducting region. Faraday's law curl E = -jωB makes it possible to express the normal of induction B on conductor surface Γ: B·n = -1/(jω) curl E·n. Galerkine's method on surface Γ is applied to this relation, which is then transformed thanks to the vector analysis formulae and Stokes' theorem:

$$\int_\Gamma wB \cdot \mathrm{n}\, d\Gamma = \frac{1}{\mathrm{j}\omega}\int_\Gamma \left(E \times n\right) \cdot grad\, w d\Gamma - \frac{1}{\mathrm{j}\omega}\int_\lambda wE \cdot d\lambda \qquad [6.23]$$

The third term of relation [6.23] is a linear integral on a contour λ located on surface Γ. It is zero on the edge between two surface elements located on the conductor surface, to ensure the continuity of the tangential component at the edge of the electric field E. This term is also zero on a symmetry plane where the condition $H \times n' = 0$ is imposed ($\phi_r = 0$) and on a symmetry plane where the condition H·n'=0 is imposed (n' is the normal to the symmetry plane). Expression (E×n) in the second term of [6.23] is expressed according to the tangential field on surface H_s, by using the surface impedance relation [6.18] and property $n \times (n \times H_s) = (n \cdot H_s)n - (n \cdot n)H_s$:

$$E \times n = E_s \times n = Z_s (n \times H_s) \times n = -Z_s\, H_s \qquad [6.24]$$

Relation [6.23] is thus written:

$$\int_\Gamma wB \cdot \mathrm{n}\, d\Gamma = \frac{1}{\mathrm{j}\omega}\int_\Gamma Z_s \operatorname{grad}_s w \cdot H_s\, d\Gamma \qquad [6.25]$$

The final formulation is obtained by using [6.21] in [6.25], and [6.25] in formulation [6.22]:

$$\int_{\Omega} \mu_0 \, \mathrm{grad} w \cdot \mathrm{grad}\, \phi_r \, d\Omega + \frac{1}{\mathrm{j}\omega} \int_{\Gamma} Z_s \, \mathrm{grad}_s \, w \cdot \mathrm{grad}_s \, \phi_r \, d\Gamma$$
$$= \int_{\Gamma} \mu_0 w H_j \cdot \mathrm{n_e} \, d\Gamma + \frac{1}{\mathrm{j}\omega} \int_{\Gamma} Z_s \, \mathrm{grad}_s \, w \cdot H_j d\Gamma \qquad [6.26]$$

The formulation in total potential is obtained by canceling the terms due to the source field H_j in the previous equation.

The passage condition $(H_2 - H_1) \times n = K$ states that the surface current density K is due to the jump of the tangential component of the field through the layer of skin depth, where $H_1 = H_s$ and H_2, which is the field inside the conductor, is assumed to be negligible with a certain depth. K and the surface density of Joule losses P_j are expressed according to field H_s calculated using potential ϕ_r thanks to [6.21] [KRA 88] [GUE 94a]:

$$K = n \times H_s, \qquad P_j = \frac{1}{2} \mathrm{Re}(Z_s) \left| H_s \right|^2 \qquad [6.27]$$

6.7.1.4. *Validity of the condition in the presence of permeable conductor*

When the permeability of the conducting region is sufficiently high and the frequency is not too high, the magnetic field in the air on conductor surface Γ is mainly normal on this surface. The numerical application of the surface impedance condition in reduced scalar potential gives bad results in this case. This problem of numerical inaccuracy is similar to the well-known permeable region problem described by the formulation in reduced potential in magnetostatics: the tangential component of the total field, which is weak, is the difference in two great numbers [PRE 92]. A solution to solve this problem consists of using the surface impedance condition in total scalar potential to describe the conductor, instead of reduced scalar potential, and including this conductor in an region of air in total scalar potential; the possible coils being in an region of air in reduced scalar potential. In [BOS 86], criteria are given to determine if the field is mainly tangential or normal on the surface of the conductors:

– if $\frac{\mu_0}{\mu} >> \frac{\delta}{L}$ and $\frac{\delta}{L} << 1$, the field is mainly tangential;

– if $\frac{\mu_0}{\mu} << \frac{\delta}{L}$ and $\frac{\mu_0}{\mu} << 1$, the field is mainly normal.

6.7.1.5. *Taking into account magnetic nonlinearities with the surface impedance condition*

6.7.1.5.1. Use of a step function B(H) curve

Preston and Deeley have developed surface impedance type methods in 3D in scalar potential for strongly saturated materials [PRE 82], [DEE 79], [DEE 86]. They have used the traditional model by Agarwal and MacLean. In this calculation model of the losses in strongly saturated steel sheets, curve B(H) is a rung (see Figure 6.9 below) [MAC 54], [AGA 59]. Under these conditions, simple loss and surface impedance formulae are obtained analytically.

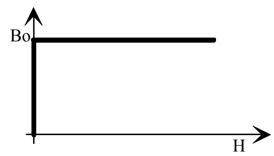


Figure 6.9. *B(H) curve in Agarwal's model*

More generally, there are two extreme cases: on the surface of the conductors, either the magnetic field is assumed to be sinusoidal, or the electric field is assumed to be sinusoidal. Actually, neither the electric field nor the magnetic field is sinusoidal, except in rare cases where it is possible to consider a 1D problem. The calculations carried out to obtain the surface impedance formulae (Z_{sH}) in the case of the sinusoidal magnetic field are detailed in [MAC 54] and [AGA 59] and in [LOW 76] in the case of the sinusoidal electric field (Z_{sE}).

$$Z_{sH} = \frac{8}{3\pi}\frac{1}{\sigma\delta}(2 + \mathrm{j}) \quad \text{and} \quad Z_{sE} = \frac{27\pi^3}{2\sqrt{5}}\frac{1}{\sigma\delta}\left(1 + \frac{4}{3\pi}\mathrm{j}\right) \qquad [6.28]$$

with $\delta = \sqrt{\dfrac{2}{\sigma\omega\mu(H_0)}} = \sqrt{\dfrac{2H_0}{\sigma\omega B(H_0)}}$

where H_0 is the peak tangent magnetic field and δ is the penetration depth of the field for a nonlinear material.

As the surface impedance depends on tangential field H_0, it is necessary to perform iterations to adjust this surface impedance, by starting with a zero field and by taking, at iteration n, the field found in the previous iteration (so-called fixed point method). In practice, 4 to 6 iterations are required. It is preferable to use the induction corresponding to the real curve B(H) rather than induction B_0 of the idealized step function curve. The results are accurate on a wider range of fields (from low to high fields). In order to ensure that the surface impedance formula is also valid in the first zone of curve B(H) (weak fields), the nonlinear and linear formulae can be weighted by a function of field H_0 [FORD 94].

6.7.1.5.2. Use of a 1D finite element model

The method presented here is more accurate than the previous one [KRA 97] [AYM 97]. It uses the solution of a 1D problem and an energy equivalence for taking into account materials with a nonlinear characteristic B(H). The solved 1D problem is the conducting plate problem having infinite thickness, subjected to a magnetic field parallel to its surface and uniform, where the plate has a nonlinear B(H) characteristic. The imposed field is such that either the magnetic field is time varying sinusoidal, or the induction is sinusoidal. The 1D problem is solved step by step over time using finite elements. For an imposed magnetic field H_1, impedance Z_s is calculated according to the surface density of active power P_S and of reactive power Q_S by formula [AYM 97]:

$$Z_S = 2\frac{dP_S + j\,dQ_S}{H_1^2} \qquad [6.29]$$

Before the principal resolution, at the beginning of this one, a characteristic $Z_s(H_1)$ is calculated. For this purpose, several impedances which correspond to various amplitudes of the imposed magnetic field H_1 are calculated. We calculate points on curve $Z_s(H_1)$ which allow all of curve B(H) to be described. Each calculated point corresponds to the resolution of the 1D problem with an amplitude H_1. The curve is obtained by interpolation between each point of calculation. During the principal resolution, this curve $Z_s(H_1)$ is directly used. As this impedance depends on the field on the surface of the conductor, iterations should be carried out, as for the method of the previous section, by the method known as the fixed point method.

6.7.1.6. *Example: transformer with its tank*

This is about a 50 Hz three-phase, three-limb distribution transformer. The magnetic circuit which is laminated is described by an region with a constant permeability with the formulation in total scalar potential. The oil inside the tank, which has the same permeability as that in the air, is described by the formulation in reduced scalar potential. As the tank is made of magnetic steel, the skin depth, approximately 3 mm at 50 Hz, is much lower than the thickness of the tank (1 cm). The field on the external side of the tank, which is an almost perfect screen, is thus considered zero. The internal surface of the tank is described by the surface impedance condition in reduced scalar potential. Calculations were carried out first with a linear B(H) characteristic of the tank then with a nonlinear one. The presence of two symmetry planes makes it possible to describe only a quarter of the device.

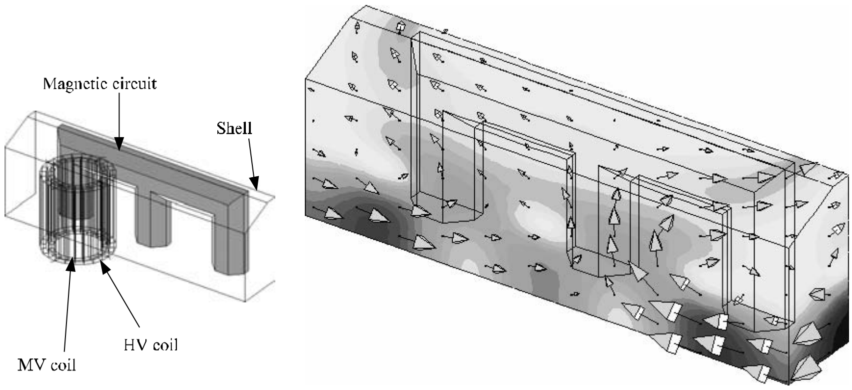


Figure 6.10. *On the left: quarter of the geometry of the transformer described in software FLUX3D (only the middle voltage (MV) and high voltage (HV) coil windings of the one of the three phases of the transformer are represented). On the right: arrows of the surface current density at a given instant, and isovalues of Joule losses in the tank (the darkest grays represent the highest losses), at rated conditions*

6.7.2. *Thin conducting regions*

6.7.2.1. *Formulations in magnetic scalar potential*

D. Rodger has proposed an interesting formulation for dealing with thin conducting regions in the case $\delta >> e$. It uses a scalar quantity "t" linked to the surface current density in the thin region and the magnetic scalar potential for the neighboring regions [ROD 87], [ROD 88], [ROD 92]. Nevertheless, the permeability of the thin region and the neighboring regions must be the same. We present in this section a more general formulation which allows the modeling of permeable thin regions while taking into account the skin effect in the thickness [GUE 94a]. This formulation, in magnetic scalar potential, requires surface elements with potential jump. The analytical solution of the conducting plate problem having a finite thickness subjected to transverse uniform fields is used [STO 74]. This analytical solution allows the surface impedances to be obtained, which will be used to obtain the formulation. This formulation was proposed in [KRA 90], [KRA 93] coupled with the boundary integral equations to take into account the neighboring regions. The formulation presented here was adapted for neighboring regions described by the finite element method [GUE 94a].

The only validity condition of this formulation is the one which states that the region must be thin: e << L where L is a characteristic length of the thin region. The materials must have a linear and isotropic permeability and conductivity.

6.7.2.1.1. Equations in the thin region

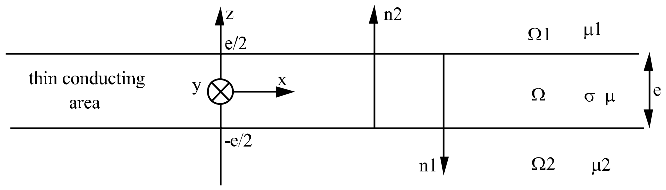


Figure 6.11. *Notations*

Let us take H_{1s} and H_{2s} as the tangential magnetic fields on both sides of the thin region Ω. The analytical solution of the one-dimensional problem of the finite thickness plate subjected to imposed fields H_{1s} and H_{2s} is given in [STO 74]. The expression of the field is written:

$$H_s(z) = \frac{1}{\mathrm{sh}ae}\left[H_{1s}\,\mathrm{sh}\left(\frac{ae}{2}+az\right)+H_{2s}\,\mathrm{sh}\left(\frac{ae}{2}-az\right)\right] \quad [6.30]$$

with $a = \dfrac{1+\mathrm{j}}{\delta}$.

Current density J is tangential to the thin region. Ampere's law curl H = J allows the expression of J to be obtained by deriving the expression of H above with respect to z:

$$J(z) = -n_1 \times \frac{\partial H_s(z)}{\partial z} = -\frac{a}{\mathrm{sh}\,ae}\,n_1 \times \left[H_{1s}\,\mathrm{ch}\left(\frac{ae}{2}+az\right)+H_{2s}\,\mathrm{ch}\left(\frac{ae}{2}-az\right)\right] \quad [6.31]$$

The electric fields E_1 on the "side 1" surface of the plate ($E_1 = E(z/2) = \rho J(z/2)$), and E_2, on side "2" ($E_2 = E(-z/2) = \rho J(-z/2)$), can thus be written:

$$E_1 = n_1 \times (\beta H_{2s} - \alpha H_{1s}) \quad [6.32]$$

$$E_2 = n_2 \times (\beta H_{1s} - \alpha H_{2s}) \quad [6.33]$$

with the complex surface impedances $\alpha = \dfrac{a}{\sigma \mathrm{th}ae}$ and $\beta = \dfrac{a}{\sigma \mathrm{sh}ae}$.

Relations [6.32] and [6.33] bind surface feature electric fields E_1 and E_2 to H_{1s} and H_{2s}, tangential magnetic fields on both sides of the thin region Ω. It is noted that relation [6.33] can be obtained from relation [6.32] by permuting indices “1” and “2”.

6.7.2.1.2. Formulation finite elements in reduced scalar potential

The following proof is similar to that in section 6.7.1.3, concerning the surface impedance condition, but differs by the thin region which comprises two borders instead of only one. Thereafter, we will focus on side “1” of the thin region. The formulation in reduced scalar potential in neighboring region Ω_1 can be written:

$$\int_{\Omega_1} \text{grad}\, w \cdot \mu_1 \,\text{grad}\, \phi_1 \, d\Omega + \int_\Gamma wB \cdot \text{n}_1 \, d\Gamma = \int_\Gamma \mu_1 w H_j \cdot \text{n}_1 \, d\Gamma \qquad [6.34]$$

The second term of [6.34] is transformed thanks to relations [6.32] and [6.33]. Thus, this term and the one corresponding to region Ω_2 allow regions Ω_1 and Ω_2 to be coupled and the thin conducting region to be taken into account. We use relation [6.23] which is valid on both borders of thin region Ω. On the border of side “1”, this relation is written:

$$\int_\Gamma wB_1 \cdot \text{n}_1 \, d\Gamma = -\frac{1}{\text{j}\omega}\int_\Gamma \left(E_1 \times n_1\right) \cdot grad\, w d\Gamma + \frac{1}{\text{j}\omega}\int_\lambda wE_1 \cdot d\lambda \qquad [6.35]$$

The line integral term disappears with the boundary conditions, and between 2 adjacent elements for the same reasons as in section 6.7.1.3. The tangential fields H_{1s} and H_{2s} are expressed according to the tangential source field H_{js} and of the reduced potential, by:

$$H_{1s} = H_{js} - grad_s\, \phi_1, \qquad H_{2s} = H_{js} - grad_s\, \phi_2 \qquad [6.36]$$

By using the electric field expression [6.32] and the propriety $\text{n} \times (\text{n} \times \text{H}_\text{s}) = (\text{n} \cdot \text{H}_\text{s})\text{n} - (\text{n} \cdot \text{n})\text{H}_\text{s}$, relation [6.35] becomes:

$$\int_\Gamma wB_1 \cdot \text{n}_1 \, d\Gamma = \frac{1}{\text{j}\omega}\int_\Gamma \text{grad}_s\, w \cdot \left(\alpha H_{1s} - \beta H_{2s}\right) d\Gamma \qquad [6.37]$$

The formulation in the thin region represented by the relation above, can now be coupled with the one of the formulation of the neighboring region Ω_1, in order to ensure the continuity of the normal component of induction B $((B_2–B_1)\cdot n_1 = 0)$ at the crossing of the interface. Having replaced relations [6.36] and [6.37] in [6.34], we obtain the final formulation:

$$\int_{\Omega_1} \mu_1 \, grad\, w \cdot grad\, \phi_1 \; d\Omega_1 + \frac{1}{j\omega}\int_\Gamma \alpha \, grad_s w \cdot grad_s \phi_1 \, d\Gamma$$
$$- \frac{1}{j\omega}\int_\Gamma \beta \, grad_s w \cdot grad_s \phi_2 \, d\Gamma = \int_\Gamma w \mu_1 H_j \cdot n_1 \mathrm{d}\,\Gamma$$
$$+ \frac{1}{j\omega}\int_\Gamma (\alpha - \beta)\mathrm{grad_s}\, w \cdot H_j \mathrm{d}\,\Gamma \qquad [6.38]$$

Equation [6.38] corresponds to side "2" of the thin region. It is necessary to associate with it the second equation which corresponds to side "1", which is deduced from [6.38] by permuting indices "1" and "2". The total scalar potential formulation is obtained by canceling the terms due to source field H_j in equation [6.38].

The passage condition $(H_2 - H_1) \times n = K$ allows the surface current impedance to be expressed as a function of the scalar potential.

$$K = n_1 \times grad_s(\phi_1 - \phi_2) \qquad [6.39]$$

The surface density of Joule losses is expressed according to the tangential components of the magnetic fields on the two sides of the thin region H_{1s} and H_{2s}, by [GUE 94a]:

$$P = \frac{1}{\sigma\delta}\frac{\frac{1}{2}\left(\left|H_{1s}\right|^2 + \left|H_{2s}\right|^2\right)(\mathrm{sh}\,2\gamma + \sin 2\gamma) - \left(H_{1s} \cdot H_{2s}^* + H_{1s}^* \cdot H_{2s}\right)(sh\gamma \cos\gamma + ch\gamma \sin\gamma)}{\mathrm{ch}\,2\gamma - \cos 2\gamma}$$

where $\gamma = \frac{e}{\delta}$ and, the surface density of reactive power is expressed according to magnetic fields H_1 and H_2 by [GUE 94a]:

$$Q = \frac{1}{\sigma\delta}\frac{\frac{1}{2}\left(\left|H_1\right|^2 + \left|H_2\right|^2\right)(\mathrm{sh}\,2\gamma - \sin 2\gamma) + \left(H_1 \cdot H_2^* + H_1^* \cdot H_2\right)(ch\gamma \sin\gamma - s\,\mathrm{h}\,\gamma \cos\gamma)}{\mathrm{ch}\,2\gamma - \cos 2\gamma}$$

Unlike the Joule losses, the reactive power is dependent on all the components of the fields. In fact, the Joule losses are calculated by integral $1/2\sigma\ |J|^2$ where current density J is tangential and is expressed according to H_{1s} and H_{2s}, whereas the reactive power is calculated by integral $1/2\ \mu\ |H|^2$ and is expressed according to the total magnetic field H. It can be shown that the variation of the normal component $H_n(z)$ has the same variation as the tangential component $H_s(z)$ given by [6.30].

In the extreme case where skin depth δ becomes very small in comparison to thickness e, impedance α tends towards surface impedance Z_S and coupling impedance β between the 2 sides becomes zero. Formulation [6.37] corresponds, in this case, to two conditions of surface impedance [6.26] on the two faces of the thin region, which decouples the 2 sides.

Inversely, in the extreme case where skin depth δ becomes very large in comparison to thickness e, impedances α and β tend towards $1/(\sigma e)$ and the power in the thin region becomes mainly resistive and the two sides are strongly coupled.

6.7.2.1.3. Edges of thin regions, line air-gaps and holes

The condition $\phi_1 - \phi_2$ = constant on a line imposes that current density K is tangential with this line. Thus, the line currents are the equipotentials of $\phi_2 - \phi_1$. When a thin region comprises an edge, this edge is a line current on which the condition "$\phi_1 - \phi_2$ = constant" must be imposed. In fact, the current density is characterized with a conservative flux in the thin region and only exists in this region. The constant is taken as equal to zero on the edge, which corresponds to confounding the degrees of freedom on the two sides: $\phi_1 = \phi_2$ [ROD 87]. Applying this condition $\phi_1 = \phi_2$ on a line allows insulating line air-gaps on a thin region to be described, for example, in the case of two jointed conducting plates, insulated between them. It is often the case of stator sheets of turbo-alternators, to reduce the losses by eddy currents. These line air-gaps must touch the edge of the thin conducting region, otherwise connectivity problems may occur [GUE 94a].

The magnetic scalar potentials can lead to problems of regions which are non-simply connected when a thin region has holes. A solution consists of describing these holes as a material of low conductivity [ROD 87].

6.7.2.1.4. Examples: transformer with its tank, conducting disk

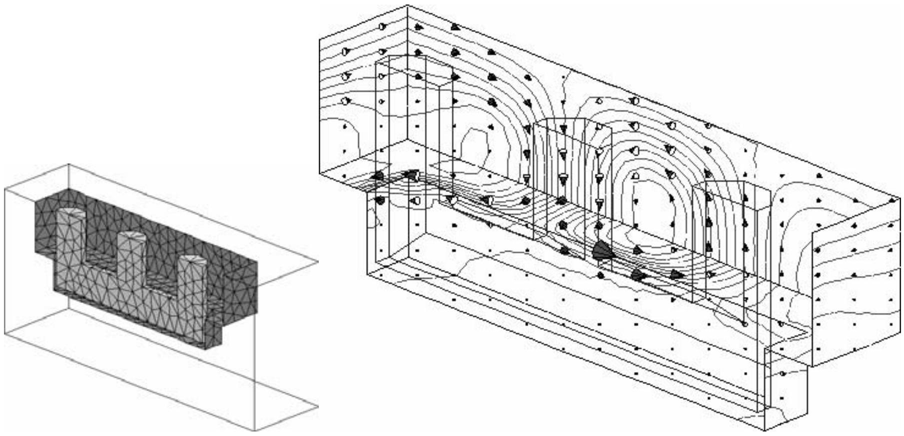


Figure 6.12. *100 MVA three-phase, three-limb transformer under overloaded conditions. Tank modeled using surface elements taking into account the skin effect along the thickness. 6 HV and LV coils are not represented. On the left, view of surface meshing of the magnetic core and the tank. On the right, lines and cones representing the surface current density in the tank*

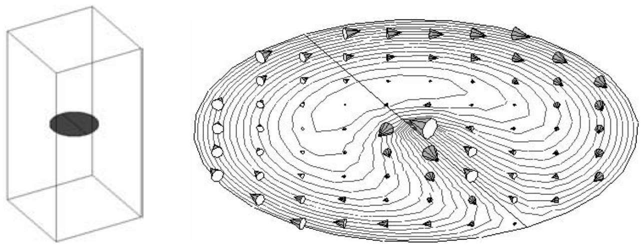


Figure 6.13. *Conducting disk with a line air-gap on a diameter, subjected to a vertical sinusoidal uniform field. On the right, lines and cones representing the surface current density*

6.7.2.1.5. Composite shells

The shields used in EMC (electromagnetic compatibility) are often made of several joined sheets of various materials. It is possible to take into account these associations of two or more layers with the formulation for thin conducting regions in magnetic scalar potential described in section 6.7.2.1 [ABA 01]. Let us consider two plates. Between these two plates, the tangential component of the magnetic field is conserved, thus the magnetic scalar potential ϕ_2 between these two plates is conserved. Let us take ϕ_1 and ϕ_3 as the potentials on the external sides of the plates. The potential ϕ_2 between the two plates is expressed by an affine combination of potentials ϕ_1 and ϕ_3. Once ϕ_2 is suppressed in the initial matrix which is dependent on the unknown variables ϕ_1, ϕ_2 and ϕ_3, a system function of only ϕ_1 and ϕ_3, which corresponds to a plate equivalent to the first two, is obtained. The method can be extended to a shield of more than two plates, by calculating the system which corresponds to a plate equivalent to two joined plates, then by calculating the system which corresponds to the equivalent plate coupled to another plate, then by continuing in a recursive way.

6.7.2.2. *AV formulation*

6.7.2.2.1. Thin regions and line regions without potential jump in AV formulation

The AV formulation without potential jump allows thin conducting regions to be described when the skin depth is significantly larger than the thickness of the thin region. The neighboring regions must be described by a formulation in potential vector A: A formulation or AV formulation.

When a thin conducting region is described by potentials A and V, the thin region is much more conducting than the neighboring regions, and the skin depth is large in comparison to thickness e, it can be shown that the current density is constant in the direction of this thickness [NAK 90]. Therefore, potentials A and V are constant in the thickness direction, the current is tangential on the surface of the thin region and the induction is normal in this surface. The validity conditions of this formulation are thus $e \ll L$, $\sigma \gg \sigma_{ext}$ and $\delta \gg e$, where L is a characteristic length of the thin region, e its thickness, σ its conductivity, σ_{ext} the conductivity of the external region and δ the skin depth in the thin region. AV formulation also allows conducting line regions to be described when the skin depth is significantly larger than the dimensions of the line region section. The surface formulation and the line formulation without potential jump are obtained from the volume formulation thanks to the methods described in sections 6.3 and 6.5.

6.7.2.2.2. Thin regions with potential jump in AV formulation

In thin regions with potential jump in AV formulation, quantities H, B, J and E can have any direction. The skin depth must be larger than the thickness of the thin region. Under these conditions, quantities H, B, J and E are assumed to be constant in region thickness. Such regions thus allow slightly conducting thin regions in very conducting volume regions to be described, as well as thin conducting regions in the air, etc. The surface formulation with potential jump is obtained using the volume formulation thanks to the general method described in section 6.4.

6.7.2.3. *Other formulations for thin conducting regions*

The formulation for thin conducting region proposed by O. Biro uses as state variables scalar quantity t in the thin region and potential vector A in the external regions. In these regions, the potential vector is used to accept the non-simply connected regions [BIR 92]. Z. Ren has used a surface element without potential jump in electric field E with edge elements [REN 90]. The external regions are taken into account by an integral method.

6.8. Thin regions in electrostatic problems, "electric harmonic problems" and electric conduction problems

For electrostatic problems, "electric harmonic problems" and electric conduction problems the state variable used is generally the electric potential. For these applications, it is possible to perform a reasoning similar to that described in section 6.2 on magnetostatics, with the magnetic scalar potentials, for thin sheets and air-gaps. For electrostatic problems and "electric harmonic problems", the surface formulation with potential jump makes it possible to describe, for example, the thin cracks located in the dielectric of capacitors. For electrostatic problems, the surface charge densities can be described by thin regions with or without potential jump. For "electric harmonic problems", the thin conducting regions with high permittivity surrounded by a vacuum can be described for the simulation of pollution on insulators.

6.9. Thin thermal regions

In thermal problems, the state variable used is generally the temperature. The surface formulation without temperature jump allows very good heat thin conducting regions to be described, i.e. having a great thermal conductivity compared to the medium where they are, for example, metal thin sections in the air, etc. In these regions, the heat flow must mainly be tangential. The surface formulation with temperature jump allows any type of thin region to be described, for example, the existing thin layers of low thermal conductivity in sandwich structures constituted by the power electronic components on their radiator. The surface densities of heat can also be described by thin regions with or without temperature jump.

6.10. References

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