
Contents

Preface	ix
Part 1. Introduction and Examples	1
Chapter 1. Overview of Inverse Problems	3
1.1. Direct and inverse problems	3
1.2. Well-posed and ill-posed problems	4
Chapter 2. Examples of Inverse Problems	9
2.1. Inverse problems in heat transfer	10
2.2. Inverse problems in hydrogeology	13
2.3. Inverse problems in seismic exploration	16
2.4. Medical imaging	21
2.5. Other examples	25
Part 2. Linear Inverse Problems	29
Chapter 3. Integral Operators and Integral Equations	31
3.1. Definition and first properties	31
3.2. Discretization of integral equations	36
3.2.1. Discretization by quadrature–collocation	36
3.2.2. Discretization by the Galerkin method	39
3.3. Exercises	42
Chapter 4. Linear Least Squares Problems – Singular Value Decomposition	45
4.1. Mathematical properties of least squares problems	45
4.1.1. Finite dimensional case	50

4.2. Singular value decomposition for matrices	52
4.3. Singular value expansion for compact operators	57
4.4. Applications of the SVD to least squares problems	60
4.4.1. The matrix case	60
4.4.2. The operator case	63
4.5. Exercises	65
Chapter 5. Regularization of Linear Inverse Problems	71
5.1. Tikhonov's method	72
5.1.1. Presentation	72
5.1.2. Convergence	73
5.1.3. The L-curve	81
5.2. Applications of the SVE	83
5.2.1. SVE and Tikhonov's method	84
5.2.2. Regularization by truncated SVE	85
5.3. Choice of the regularization parameter	88
5.3.1. Morozov's discrepancy principle	88
5.3.2. The L-curve	91
5.3.3. Numerical methods	92
5.4. Iterative methods	94
5.5. Exercises	98
Part 3. Nonlinear Inverse Problems	103
Chapter 6. Nonlinear Inverse Problems – Generalities	105
6.1. The three fundamental spaces	106
6.2. Least squares formulation	111
6.2.1. Difficulties of inverse problems	114
6.2.2. Optimization, parametrization, discretization	114
6.3. Methods for computing the gradient – the adjoint state method	116
6.3.1. The finite difference method	116
6.3.2. Sensitivity functions	118
6.3.3. The adjoint state method	119
6.3.4. Computation of the adjoint state by the Lagrangian	120
6.3.5. The inner product test	123
6.4. Parametrization and general organization	123
6.5. Exercises	125
Chapter 7. Some Parameter Estimation Examples	127
7.1. Elliptic equation in one dimension	127
7.1.1. Computation of the gradient	128
7.2. Stationary diffusion: elliptic equation in two dimensions	129

7.2.1. Computation of the gradient: application of the general method	132
7.2.2. Computation of the gradient by the Lagrangian	134
7.2.3. The inner product test	135
7.2.4. Multiscale parametrization	135
7.2.5. Example	136
7.3. Ordinary differential equations	137
7.3.1. An application example	144
7.4. Transient diffusion: heat equation	147
7.5. Exercises	152
Chapter 8. Further Information	155
8.1. Regularization in other norms	155
8.1.1. Sobolev semi-norms	155
8.1.2. Bounded variation regularization norm	157
8.2. Statistical approach: Bayesian inversion	157
8.2.1. Least squares and statistics	158
8.2.2. Bayesian inversion	160
8.3. Other topics	163
8.3.1. Theoretical aspects: identifiability	163
8.3.2. Algorithmic differentiation	163
8.3.3. Iterative methods and large-scale problems	164
8.3.4. Software	164
Appendices	167
Appendix 1	169
Appendix 2	183
Appendix 3	193
Bibliography	205
Index	213