
Contents

Preface	ix
Introduction	xiii
Chapter 1. Reminders on Linear Algebra	1
1.1. Linear spaces	1
1.1.1. Definition	1
1.1.2. Algebraic basis of linear spaces	2
1.2. Linear applications	3
1.2.1. Definition	3
1.2.2. Injections - surjections - identification	4
1.2.3. Algebraic duality	5
1.2.4. Space of functions zero everywhere except in a finite number of points	7
1.3. Partition, quotient linear space	8
1.3.1. Partition and equivalence relation	8
1.3.2. Quotient linear space	11
1.4. Skew-symmetric multi-linear maps	13
1.4.1. Permutations	13
1.4.2. Alternating multi-linear map	15
1.4.3. Identification of bilinear maps	17
Chapter 2. Construction of Exterior Algebras	19
2.1. Existence	20
2.1.1. Construction theorem statement	20
2.1.2. Demonstration for $p = 2$	20
2.1.3. Demonstration for $p > 2$	23
2.2. Uniqueness of exterior algebra with degree p	28
2.3. Some remarks	28

Chapter 3. Exterior Product Symbol	31
3.1. Exterior product symbol \wedge in exterior algebras	31
3.1.1. Construction principle	31
3.1.2. Properties	34
3.1.3. Grassmann's algebra	36
3.2. Symbol of exterior product \wedge^* between forms	38
3.2.1. Construction principle	38
3.2.2. Elementary properties of the symbol of exterior product between forms	39
3.2.3. Characterization of linear independent families	42
Chapter 4. Bases of Exterior Algebras	45
4.1. Construction of algebraic bases	45
4.1.1. Generating a set of exterior algebras	45
4.1.2. Linearly independent family of exterior algebras	47
4.1.3. Finite dimensional Grassmann's algebra	49
4.2. Identification of the dual of $\Lambda^p E$	50
4.2.1. Identification of alternating 2-linear forms over \mathbb{R}^3	50
4.2.2. Basis of alternating p -linear forms over E^p	51
4.2.3. Skew-symmetric form and dual space	52
4.2.4. Structure of the dual: finite dimensional case	54
Chapter 5. Determinants	55
5.1. Determinant of vector families	55
5.1.1. Definition and calculation	55
5.1.2. Components of the exterior product	57
5.1.3. Properties of determinants	59
5.2. Practical calculation of determinants	60
5.2.1. Transposed determinant	60
5.2.2. Matrix illustration of determinant calculation	61
5.2.3. Laplace expansion along a row or a column	62
5.2.4. Expansion by blocks	64
5.2.5. Expansion by exploiting the associativity	65
5.3. Solution of linear systems	68
5.3.1. Cramer formulas	68
5.3.2. Rouché-Fontené theorem	69
5.3.3. Example	73
Chapter 6. Pseudo-dot Products	77
6.1. Quadratic forms and symmetric bilinear forms	77
6.1.1. Definition - correspondence	77
6.1.2. Sign of the forms: on the possibility of restricting the forms to subspaces	80

6.1.3. Representation via nondegenerate forms	82
6.2. Orthogonality	85
6.2.1. Definition - elementary properties	85
6.2.2. Existence and construction of orthonormal bases	88
6.2.3. Signature of a nondegenerate quadratic form	93
Chapter 7. Pseudo-Euclidean Algebras	95
7.1. Pseudo-dot product over $\Lambda^p E$	95
7.1.1. Construction	95
7.1.2. Calculations and applications	98
7.2. Applications: volume and infinitesimal volumes	101
7.2.1. Measure of a parallelotope	102
7.2.2. Measure of a simplex	104
7.2.3. Usage of coordinates of the exterior product vector	107
7.2.4. “Infinitesimal” measure of classical objects	108
Chapter 8. Divisibility and Decomposability	111
8.1. Contraction product	111
8.1.1. Definition	111
8.1.2. Calculation formulas	112
8.2. Divisibility by a k -blade	115
8.2.1. General case	115
8.2.2. Exterior vector division	117
8.3. Decomposability	119
Chapter 9. H-conjugation and Regressive Product	121
9.1. Introduction to H -conjugation	121
9.1.1. Definition	121
9.1.2. Elementary properties of the H -conjugation	127
9.1.3. On the confusion regarding the vector cross product in three-dimensional spaces	130
9.1.4. Demonstration of determinant calculation rules	132
9.2. Regressive product	133
9.2.1. Definition	133
9.2.2. Some calculation formulas	136
Chapter 10. Endomorphisms of Exterior Algebras	141
10.1. Constructible endomorphisms	141
10.1.1. Construction of an endomorphism over $\Lambda^p E$	141
10.1.2. Invariants of endomorphism families	147
10.1.3. Applications of invariants of an endomorphism	148
10.1.4. Conjugated endomorphism	153

10.2. Decomposition of endomorphisms of exterior algebras	155
10.2.1. Examination of $\mathcal{L}(\Lambda^2\mathbb{R}^3)$	156
10.2.2. Constructibility of endomorphisms of $\Lambda^{n-1}\mathbf{E}$	159
10.2.3. Laplace inversion formula	162
Chapter 11. $\Lambda^2\mathbf{E}$ Algebra	165
11.1. Correspondence between a skew-symmetric operator and $\Lambda^2\mathbf{E}$ elements	165
11.1.1. Elementary reminders on operators' symmetries	165
11.1.2. Lack of diagonalization in pseudo-Euclidean spaces	167
11.1.3. Isomorphism between skew-symmetric operators and $\Lambda^2\mathbf{E}$ elements	169
11.2. Decomposability within $\Lambda^2\mathbf{E}$	171
11.2.1. Simple decomposability	171
11.2.2. p -Decomposability	173
11.2.3. Interpretation in terms of operators	178
11.2.4. Pseudo-orthodecomposability	180
Bibliography	189
Index	191