

Table of Contents

Preface	xvii
Part 1. Level 1	1
Chapter 1. The Basics of Linear Elastic Behavior	3
1.1. Cohesion forces	4
1.2. The notion of stress	6
1.2.1. Definition	6
1.2.2. Graphical representation	7
1.2.3. Normal and shear stresses	8
1.3. Hooke's law derived from a uniaxially applied force	9
1.3.1. The stretch test	9
1.3.2. Linear mechanical behavior	12
1.3.3. Elastic mechanical behavior	12
1.3.4. Interpretation of the test at a macroscopic level	13
1.3.5. Interpretation of the test at a mesoscopic level	13
1.3.6. Interpretation of the test at a microscopic level	16
1.3.7. Summary	18
1.4. Plane state of stresses	20
1.4.1. Definition	20
1.4.2. Behavior relationships for state of plane stresses	22
1.4.2.1. Case 1: simple tension along \vec{x}	22
1.4.2.2. Case 2: simple tension along \vec{y}	24
1.4.2.3. Case 3: pure shear	25
1.4.2.4. Complete state of stress (superposition)	28
1.4.3. Summary	35
1.5. Particular case of straight beams	36
1.5.1. Preliminary observations	36

1.5.1.1. Geometric characteristics	36
1.5.1.2. Resultant force and moment for cohesion forces	36
1.5.2. Effects linked to the resultant forces and moments	38
1.5.2.1. Normal resultant	40
1.5.2.2. Shear resultant \mathcal{T}_y	42
1.5.2.3. Shear resultant \mathcal{T}_z	43
1.5.2.4. Torsion moment $\mathcal{M}t$	45
1.5.2.5. Bending moment $\mathcal{M}f_y$	46
1.5.2.6. Bending moment $\mathcal{M}f_z$	48
Chapter 2. Mechanical Behavior of Structures: An Energy Approach	51
2.1. Work and energy	51
2.1.1. Elementary work developed by a force.	51
2.1.2. Elementary work developed by a moment.	52
2.2. Conversion of work into energy	53
2.2.1. Potential energy of deformation	53
2.2.2. Potential energy for a spring	55
2.3. Some standard expressions for potential deformation energy.	58
2.3.1. Deformation energies in a straight beam	58
2.3.1.1. Traction (or compression).	58
2.3.1.2. Torsion	61
2.3.1.3. Pure bending (xy plane).	65
2.3.1.4. Plane bending (xy plane)	69
2.3.2. Deformation energy under plane stresses.	74
2.3.2.1. Case 1: dF_x (Figure 2.17)	74
2.3.2.2. Case 2: dF_x then dF_y (Figure 2.18).	75
2.3.2.3. Case 3: dF_x then dF_y followed by dF_{xy} (Figure 2.19).	76
2.3.2.4. Different expressions for potential energy: quadratic forms.	77
2.4. Work produced by external forces on a structure	81
2.4.1. Beam under plane bending subjected to two forces.	82
2.4.1.1. Example 1	82
2.4.1.2. Example 2	93
2.4.2. Beam in plane bending subject to “n” forces	99
2.4.3. Generalization to any structure.	103
2.4.3.1. Structure loaded by two forces \vec{F}_1 and \vec{F}_2	103
2.4.3.2. Structure loaded by “n” forces $\vec{F}_1, \dots, \vec{F}_n$	105
2.4.3.3. A search for real displacements on a loaded structure	107
2.4.4. Summary.	112
2.5. Links of a structure with its surroundings	113
2.5.1. Example	113

2.5.2. Generalization	118
2.5.2.1. Structures with rigid-body movements	118
2.5.2.2. “Properly linked” structure	119
2.6. Stiffness of a structure	119
2.6.1. Preliminary note	119
2.6.2. Stiffness matrix	121
2.6.3. Examples	121
2.6.3.1. Example: beam under plane bending loaded by two forces	121
2.6.3.2. Example: beam under plane bending loaded by a force and a moment	123
2.6.3.3. Generalization.	125
2.6.4. Influence of the positioning.	125
2.6.4.1. Example: bar working under traction-compression.	125
2.6.4.2. Example: stiffness matrix of a beam structure under plane bending.	129
2.6.4.3. Isostatism and hyperstatism.	135
2.6.5. Deformation energy and stiffness matrix.	139
2.6.5.1. Example: beam from section 2.6.3.1	139
2.6.5.2. Generalization.	140
Chapter 3. Discretization of a Structure into Finite Elements.	143
3.1. Preliminary observations	143
3.1.1. Problem faced.	143
3.1.2. Practical obtaining of the deformation energy for a complex structure	144
3.1.3. Local and global coordinates	147
3.1.3.1. Definition	147
3.1.3.2. Application to the elements of the structure	149
3.1.3.3. Summary.	152
3.2. Stiffness matrix of some simple finite elements.	153
3.2.1. Truss element loaded under traction (or compression).	153
3.2.1.1. Summary: pure traction (or compression) exerted on a beam.	153
3.2.1.2. Stiffness matrix	153
3.2.1.3. Beam element under torsion	160
3.2.1.4. Torsion loading on a beam element	161
3.2.1.5. Stiffness matrix	161
3.2.3. Beam element under plane bending	168
3.2.3.1. Summary: plane bending of a beam.	168
3.2.3.2. Stiffness matrix	168
3.2.4. Triangular element for the plane state of stresses	178
3.2.4.1. Preliminary comment	178
3.2.4.2. Definition of the element	178

3.2.4.3. Form of the displacement functions	179
3.2.4.4. Determination of the stiffness matrix	181
3.2.4.5. Example	184
3.2.4.6. Performance improvement of the element	187
3.2.4.7. Summary	189
3.3. Getting the global stiffness matrix of a structure	191
3.3.1. Objective	191
3.3.2. Mechanism of the assembly of elementary matrices	191
3.3.2.1. Example 1	191
3.3.2.2. Example 2	198
3.3.1. Introduction	201
3.4. Resolution of the system $\{F\} = [K] \bullet \{d\}$	203
3.4.1. Linkage conditions	203
3.4.2. Generalization of the method	205
3.5. Different types of finite elements available in industrial software	207
Chapter 4. Applications: Discretization of Simple Structures	209
4.1. Stiffness matrix of a spring	209
4.1.1. Helical spring	209
4.1.2. Spiral spring	211
4.2. Assembly of elements	213
4.2.1. Example 1	213
4.2.2. Example 2	217
4.2.3. Example 3	222
4.2.4. Assembly of a truss element and a beam element under simple plane bending	228
4.3. Behavior in the global coordinate system	232
4.3.1. Plane assembly of two truss elements	232
4.4. Bracket	246
4.4.1. Objectives	246
4.4.2. Modelizing	247
4.4.2.1. Definition of the beam element	247
4.4.2.2. Model using wires	248
4.4.2.3. Geometric properties of the beams	249
4.4.2.4. Support conditions	251
4.4.2.5. Loading	251
4.4.3. Calculation of the elementary stiffness matrix in the global system	252
4.4.4. Assembly of the global stiffness matrix $[K]_{str}$	256
4.4.5. Establishing the linkage and loading conditions	259
4.4.6. Resolution of the linear system $\{F\}_{str} = [K]_{str} \bullet \{d\}_{str}$	260
4.4.7. Additional study of the behavior of the bracket	262
4.4.7.1. Internal linking forces on each of the isolated elements	262

4.4.7.2. Normal stresses	265
4.4.8. Using computing software	267
Part 2. Level 2	269
Chapter 5. Other Types of Finite Elements	271
5.1. Return to local and global coordinate systems.	271
5.1.1. Transfer matrix	271
5.1.2. Summary.	273
5.2. Complete beam element (any loading case)	274
5.2.1. Preliminary comments.	274
5.2.2. Obtaining the stiffness matrix in the local coordinate system.	276
5.2.3. Improvement in performances of this beam element	283
5.2.3.1. Supplementary deformation due to the shear	283
5.2.3.2. Combination of bending and torsion	285
5.2.4. Summary.	288
5.3. Elements for the plane state of stress	291
5.3.1. Triangular element	291
5.3.1.1. Preliminary comments.	291
5.3.1.2. Summary.	292
5.3.2. Quadrilateral element in plane state of stress	295
5.3.2.1. Rectangular element	295
5.3.2.2. Quadrilateral element	298
5.3.2.3. Summary.	298
5.4. Plate element	300
5.4.1. Preliminary notes.	300
5.4.2. Resultant forces and moments for cohesion forces	302
5.4.3. Plate element in bending	305
5.4.3.1. Rectangular element	305
5.4.3.2. Triangular element	308
5.4.4. Complete plate element	310
5.5. Elements for complete states of stresses	315
5.5.1. Preliminary notes.	315
5.5.2. Solid tetrahedric element	318
5.5.3. Solid parallelepipedic element	321
5.6. Shell elements.	327
5.6.1. Preliminaries	327
5.6.2. Specific case of axisymmetric shells	328
5.6.3. Axisymmetric shell element with axisymmetric boundaries	329

Chapter 6. Introduction to Finite Elements for Structural Dynamics	331
6.1. Principles and characteristics of dynamic study	332
6.1.1. Example 1	332
6.1.1.1. Description of motion	332
6.1.1.2. Dynamic behavior relation	333
6.1.1.3. Elastic behavior relation.	333
6.1.1.4. Equation of motion.	334
6.1.2. Example 2	338
6.1.2.1. Dynamic behavior relation	338
6.1.2.2. Elastic behavior relation.	339
6.1.2.3. Equations of motion	340
6.1.2.4. Eigenmodes of vibration	340
6.2. Mass properties of beams	346
6.2.1. Finite beam element in dynamic bending plane	346
6.2.2. Discretization of a beam for dynamic bending	350
6.2.3. Other types of dynamic behaviors of a beam	357
6.2.3.1. Truss element in dynamic tension-compression.	357
6.2.3.2. Beam element with circular cross-section in dynamic torsion	359
6.3. Generalization.	363
6.4. Summary.	364
Chapter 7. Criteria for Dimensioning	365
7.1. Designing and dimensioning	365
7.2. Dimensioning in statics	370
7.2.1. The two types of criteria	370
7.2.2. Elasticity limit criterion	373
7.2.2.1. Intrinsic surface.	373
7.2.2.2. Complete state of stresses.	375
7.2.2.3. Von Mises criterion	376
7.2.3. Non-rupture criterion	382
7.2.3.1. Brittle materials.	382
7.2.3.2. Elastoplastic materials.	383
7.3. Dimensioning in fatigue.	393
7.3.1. Fatigue phenomenon.	393
7.3.2. Fatigue test	394
7.3.3. Modeling of the fatigue	397
7.3.3.1. Modeling of dynamic loading	397
7.3.3.2. Corresponding fatigue test	398
7.3.4. Estimation of fatigue strength	399
7.3.4.1. Case of a simple wavy load.	399
7.3.4.2. Case of multiple wavy loads	402

Chapter 8. Practical Aspects of Finite Element Modeling	407
8.1. Use of finite element software	407
8.1.1. Introduction	407
8.1.2. Summary tables of the properties of elements	408
8.1.3. Connection between elements of different types	415
8.1.3.1. Introduction	415
8.1.3.2. Example 1	416
8.1.3.3. Example 2	417
8.1.3.4. Example 3	418
8.1.3.5. Conclusion.	419
8.1.4. Other practical aspects.	420
8.1.4.1. Symmetric structures	421
8.1.4.2. “Floating” models	421
8.1.4.3. Modeling of fabricated welded structures	426
8.1.4.4. “Nonlinear geometric” behavior	429
8.2. Example 1: machine-tool shaft.	432
8.2.1. Simulation exercise	432
8.2.2. Data.	433
8.2.3. Successive steps of modeling.	434
8.2.3.1. Definition of the loading	434
8.2.3.2. Linkings of the structure at boundaries	435
8.2.3.3. Finite element discretization of shaft (2)	437
8.3. Example 2: thin-walled structures	440
8.3.1. Model based on beam elements	441
8.3.1.1. Methodology	441
8.3.1.2. Junctions of centerlines	442
8.3.1.3. Relieving of linkings.	444
8.3.2. Model in plate elements.	448
8.3.3. Model in beam and plate elements	449
8.4. Example 3: modeling of a massive structure.	450
8.4.1. Problem	450
8.4.2. Steps of modeling	451
8.4.2.1. Structural parts	451
8.4.2.2. Choosing the type of finite element	451
8.4.2.3. Forces applied on the body	451
8.4.2.4. Boundary conditions.	452
8.4.2.5. Taking symmetries into account	454
8.4.2.6. Other aspects of the modeling	455
8.4.3. Comments on the validity of the model.	456
8.5. Summary of the successive modeling steps	457
8.5.1. Preliminary analysis	457
8.5.2. Model verification and validation	458

8.5.2.1. Before calculation	458
8.5.2.2. After calculation	459
8.5.3. Corresponding use of the software	460
Part 3. Supplements	463
Chapter 9. Behavior of Straight Beams	465
9.1. The “straight beam” model	466
9.1.1. Definition	466
9.1.2. Main or “principal” axis of a cross-section	466
9.1.3. Applied loadings	468
9.1.4. Cohesion force and moment on a current cross-section	469
9.1.4.1. Equilibrium of the beam.	470
9.1.4.2. Defining the resultant force and moment for cohesion forces.	471
9.1.4.3. Projections of cohesive resultant force and moment on local axis.	471
9.1.5. Hypothesis of the beam theory	474
9.1.5.1. Hypothesis on stresses.	474
9.1.5.2. Hypothesis on deformations	478
9.1.6. Microscopic equilibrium	479
9.2. Mesoscopic equilibrium or equilibrium extended to a whole cross-section	482
9.3. Behavior relations and stresses.	486
9.3.1. Normal resultant	486
9.3.1.1. Definition	486
9.3.1.2. Deformation of an elementary slice of beam.	487
9.3.1.3. Stresses on a cross-section	488
9.3.2. Torsional loading	490
9.3.2.1. Definition	490
9.3.2.2. Deformation of an elementary beam slice	491
9.3.2.3. Simple case of a circular section.	492
9.3.2.4. Case of a non-circular cross-sectional shape.	495
9.3.2.5. Torsion characteristics for some particular cross-sectional shapes	510
9.3.2.6. Torsion with constrained warping	513
9.3.3. Pure bending	514
9.3.3.1. Pure bending in the specific case of a beam with a plane of symmetry	514
9.3.3.2. The general case of pure bending	525
9.3.4. Plane bending with shear resultant	530
9.3.4.1. Definition	530
9.3.4.2. Displacement field	531

9.3.4.3. Analysis of displacement $\eta(x, y, z)$	534
9.3.4.4. Shear stresses	537
9.3.4.5. Behavior relation for the shear resultant	538
9.3.4.6. Application: case of a rectangular section	541
9.3.4.7. Values of the shear coefficient k_y and shear section S_{ry} for some section shapes	543
9.3.4.8. Summary.	545
9.3.5. Any loading	549
9.4. Application: example of detailed calculation of the resultant forces and moments of cohesive forces	551
9.4.1. Preliminary static analysis	551
9.4.2. Resultant force and moment on every cross-section	553
Chapter 10. Additional Elements of Elasticity	563
10.1. Reverting to the plane state of stresses	563
10.1.1. Influence of the coordinate system.	563
10.1.2. Principal directions and stresses	566
10.1.3. Mohr graphical representation	568
10.1.4. Summary.	575
10.1.5. Some remarkable plane states of stresses with their Mohr representation.	576
10.1.5.1. Case 1.	576
10.1.5.2. Case 2.	577
10.1.5.3. Case 3.	581
10.1.5.4. Case 4: cylindrical vessel under pressure	582
10.1.5.5. Numerical example.	586
10.1.6. Experimental evaluation of deformations to define stresses	588
10.1.7. Deformation energy in principal axes	593
10.2. Complete state of stresses	593
10.2.1. Principal directions and stresses	593
10.2.2. Stresses in any \vec{x} , \vec{y} , \vec{z} axes	594
10.2.3. Deformations	597
10.2.4. Behavior relations	599
10.2.5. Strain potential energy.	604
10.2.6. Summary.	609
10.2.7. Components of the strain potential energy	612
10.2.7.1. Strain energy without distortion	612
10.2.7.2. Distortion strain energy	616
10.2.7.3. Summary	617

Chapter 11. Structural Joints	619
11.1. General information on connections by means of cylindrical fasteners	620
11.1.1. Contact pressure	620
11.1.2. General information on riveting	622
11.1.2.1. Transmission of mechanical loads in riveted joints	622
11.1.2.2. Functioning of a rivet	623
11.1.3. General information on bolted joints	624
11.1.3.1. Transmission of mechanical loads in a bolted joint	624
11.1.3.2. Functioning of threaded fasteners	625
11.1.4. Deterioration of riveted and bolted joints	626
11.1.4.1. Rupture of fasteners	626
11.1.4.2. Bearing	627
11.1.4.3. Spacing of fasteners	629
11.2. Bolted joint	631
11.2.1. Simplified case where the tightening is neglected	631
11.2.1.1. Hypotheses	631
11.2.1.2. Model of joining interface	634
11.2.1.3. Forces on each fastener	637
11.2.1.4. Resistance criteria	648
11.2.1.5. Summary	649
11.2.1.6. Example	651
11.2.2. Case of pre-tightening	654
11.2.2.1. Tightening torque	654
11.2.2.2. Behavior of a bolted joint with pre-tightening	655
11.2.2.3. Summary	663
11.2.2.4. Example	665
11.3. Riveted joint	666
11.3.1. Hypotheses	666
11.3.2. Characteristics of the modeled joining interface	666
11.3.3. Forces on each attachment	667
11.3.4. Graphic representation of the shear stresses	668
11.3.5. Summary	670
11.4. Welded joints	671
11.4.1. Preliminary observations and hypotheses	671
11.4.1.1. State of stresses in a weld bead	671
11.4.1.2. Dimensioning criterion	672
11.4.2. Determination of the stresses in the weld bead cross-section	673
11.4.2.1. Statutory aspect	673
11.4.2.2. Definition of a model for the dimensioning of a weld interface	679
11.4.2.3. Stresses on each “equivalent” bead	680
11.4.2.4. Stresses σ_n , τ_ℓ , τ_t in the bead cross-sections	683

11.4.3. Summary	686
11.4.4. Example	688
Chapter 12. Mathematical Prerequisites	691
12.1. Matrix calculus	691
12.1.1. General information	691
12.1.1.1. Definition of a matrix	691
12.1.1.2. Symmetric matrix.	692
12.1.1.3. Transposition of a matrix [a]	692
12.1.2. Matrix operations	692
12.1.2.1. Addition of two matrices	692
12.1.2.2. Product of a matrix by a scalar	694
12.1.2.3. Product of two matrices	694
12.1.2.4. Inverse of a matrix	695
12.1.3. Quadratic form	696
12.1.4. Eigenvalues and eigenvectors of a matrix	697
12.1.4.1. Eigenvalues	697
12.1.4.2. Eigenvectors	697
12.2. Change in orthonormal coordinate system	698
12.2.1. Case of coplanar coordinate systems	698
12.2.2. Cases of any general coordinate systems	699
Appendix A. Modeling of Common Mechanical Joints	703
A.1. Definition	703
A.1.1. Monolithic unit.	703
A.1.2. Joints.	703
A.1.3. Perfect joints	704
A.2. Common standardized mechanical joints (ISO 3952)	704
Appendix B. Mechanical Properties of Materials	711
B.1. Mechanical properties of some materials used for structures.	711
B.1.1. Steels and casting	711
B.1.2. Non-ferrous metals	712
Appendix C. List of Summaries	713
Bibliography	717