From Euclidean to Hilbert Spaces

Introduction to Functional Analysis and its Applications

Edoardo Provenzi

Color Section

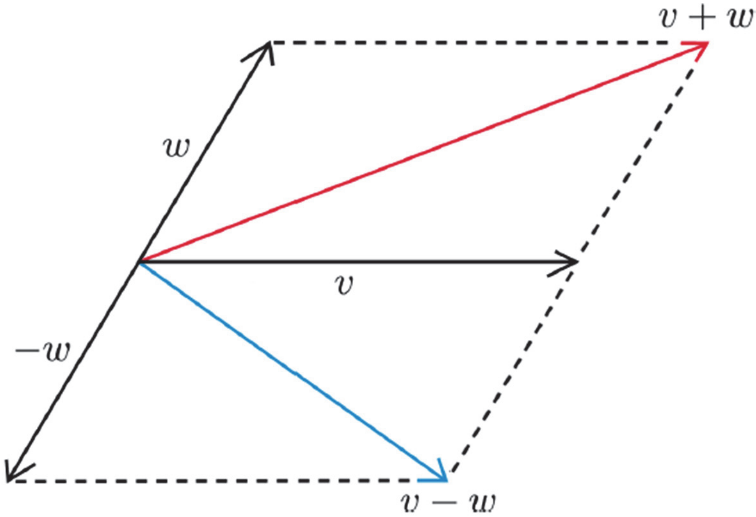

Figure 1.1. *Parallelogram law in R2: The sum of the squares of the two diagonal lines is equal to two times the sum of the squares of the edges v and w*

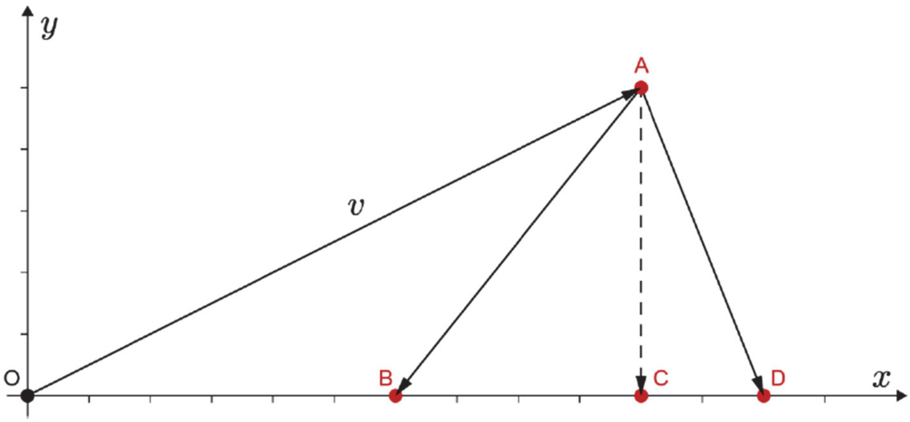

Figure 1.2. *Orthogonal projection $P_x v = \vec{OC}$ and diagonal projections $\vec{OB}$ and $\vec{OD}$ of a vector in $v \in \mathbb{R}^2$ onto the x axis.*

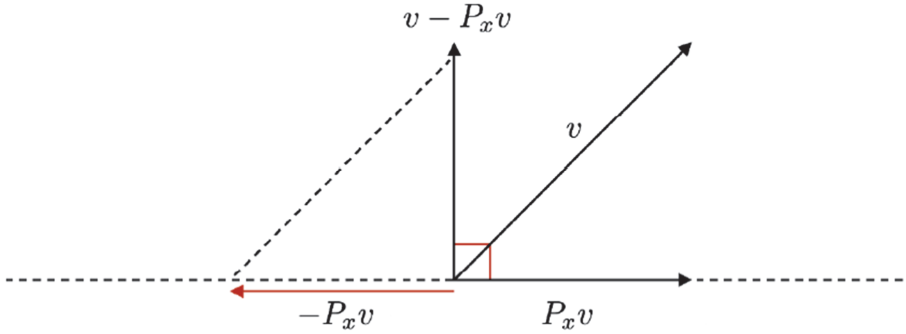

Figure 1.3. *Visualization of property 2 in* $\mathbb{R}^2$.

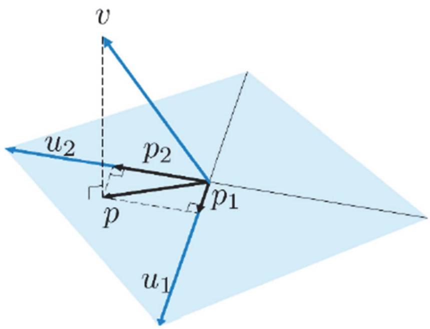

Figure 1.4. *Orthogonal projection* p *of a vector in* $\mathbb{R}^3$ *onto the plane produced by two unit vectors*

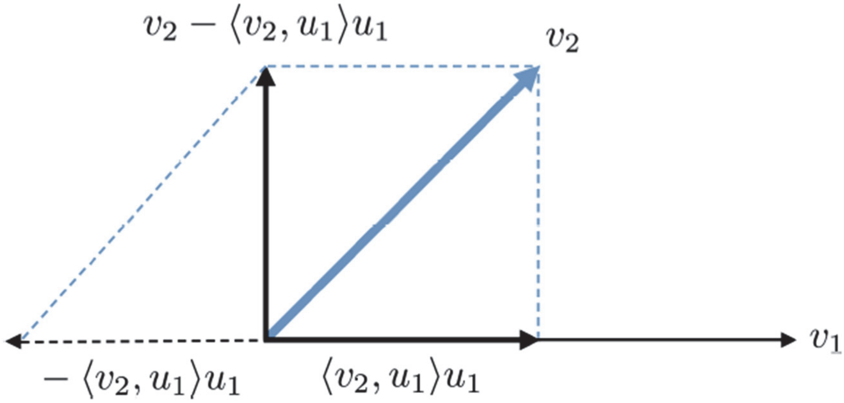

Figure 1.5. *Illustration of the second step in the Gram-Schmidt orthonormalization process*

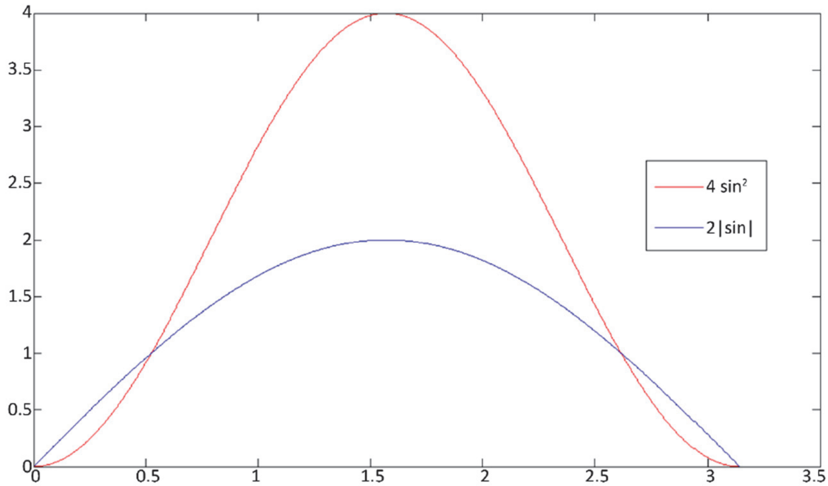

Figure 2.4. *Difference between the sine functions representing the spectrum values of the first and second derivative operators between 0 and π*

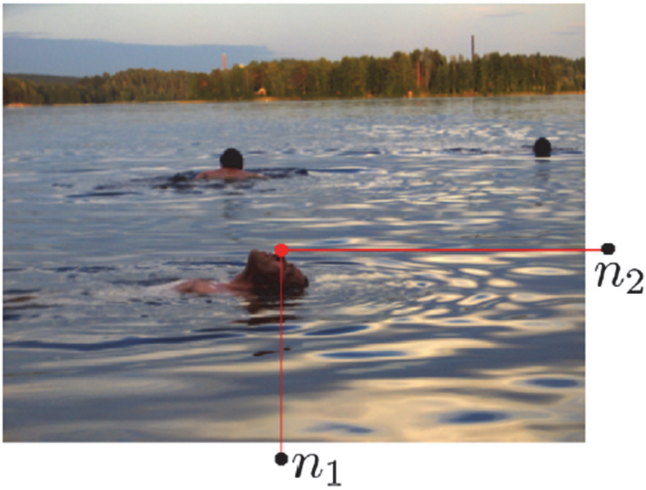

Figure 2.5. *The two coordinates of a pixel,* n_1, n_2, *in a digital image (image source: author)*

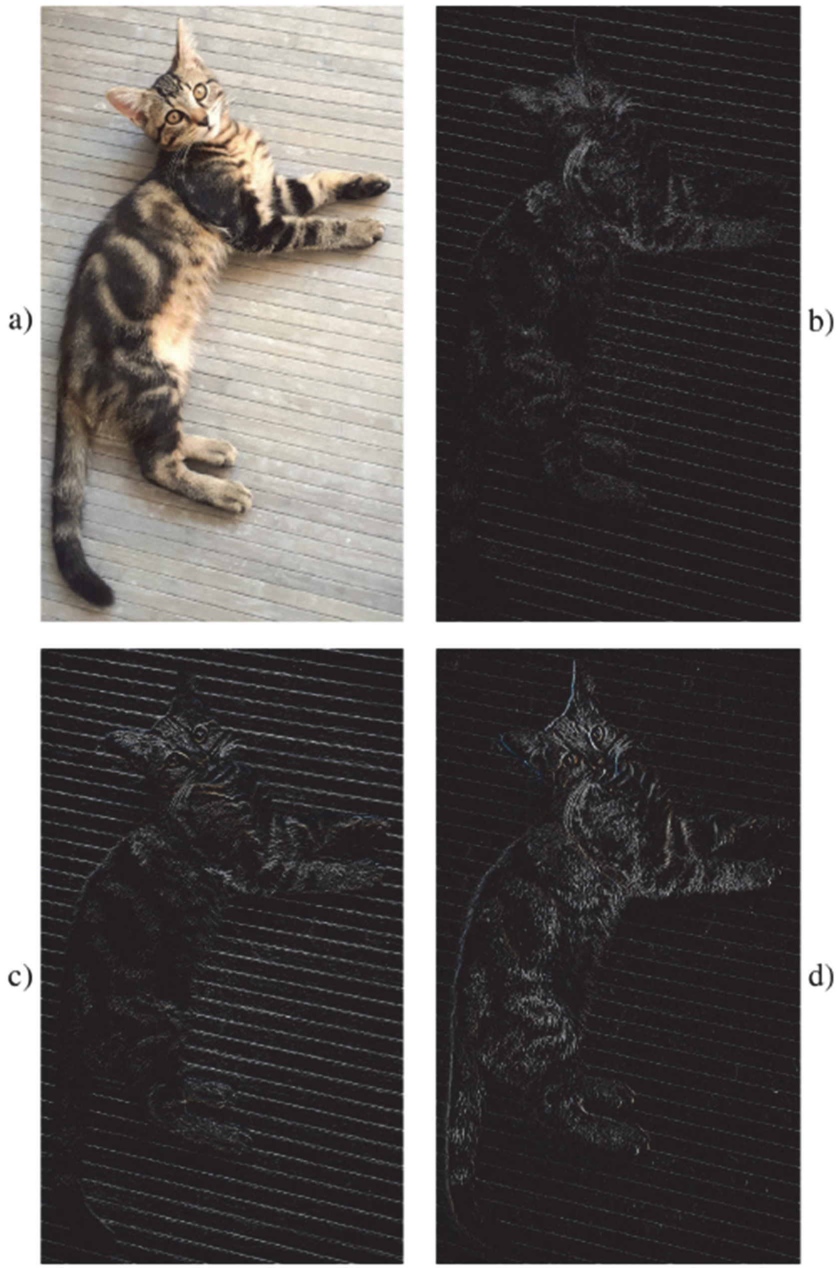

Figure 2.6. *a) Original image of Panko; b) image after Laplacian filter; c) image filtered using a gradient in the vertical direction; d) image filtered using a gradient in the horizontal direction (image source: author)*

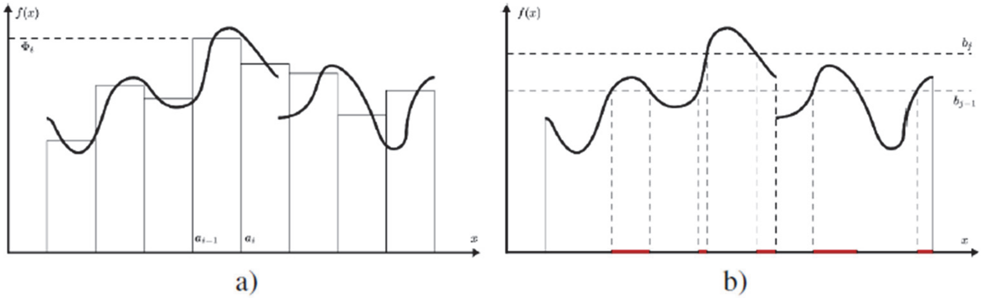

Figure 3.1. *Riemann and Lebesgue integration*

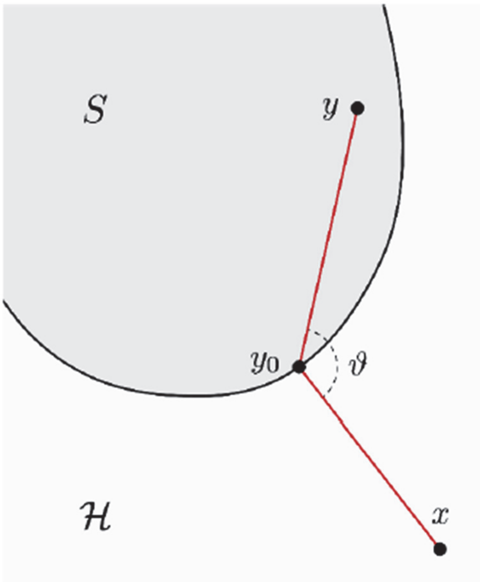

Figure 5.1. *Two-dimensional geometric visualization of the property verified by the projection onto a closed, convex and proper subset of H*

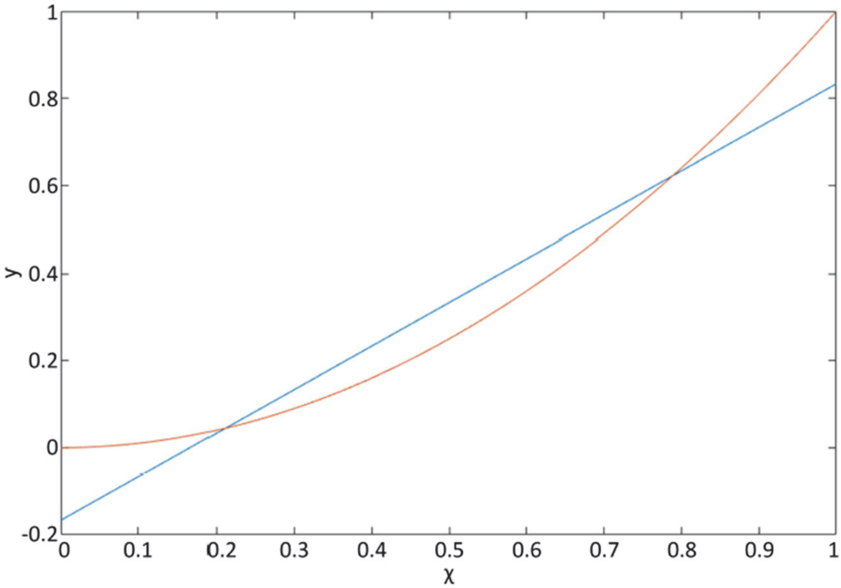

Figure 6.1. *The line of equation* $y = x - \frac{1}{6}$ *(shown in blue) is the best approximation of the parabola with equation* $y = x^2$ *(in red) with respect to the Hilbert norm of* $L^2[0,1]$.