Contents

Preface ix

List of Notations xiii

Chapter 1. Branching Processes in Varying Environment 1

1.1. Introduction 1
1.2. Extinction probabilities 6
1.3. Almost sure convergence 12
1.4. Family trees 15
1.4.1. Construction of the Geiger tree 20
1.4.2. Construction of the size-biased tree T^* 21
1.5. Notes 22

Chapter 2. Branching Processes in Random Environment 25

2.1. Introduction 25
2.2. Extinction probabilities 28
2.3. Exponential growth in the supercritical case 30
2.4. Three subcritical regimes 35
2.5. The strictly critical case 43
2.6. Notes 45

Chapter 3. Large Deviations for BPREs 47

3.1. Introduction 47
3.2. A tail estimate for branching processes in a varying environment 50
3.3. Proof of Theorem 3.1 55
3.4. Notes 59

Chapter 4. Properties of Random Walks 61

4.1. Introduction 61
4.2. Sparre-Andersen identities 62
4.3. Spitzer identity 69
4.4. Applications of Sparre-Andersen and Spitzer identities 70
4.4.1. Properties of ladder epochs and ladder heights 70
4.4.2. Tail distributions of ladder epochs 75
4.4.3. Some renewal functions 76
4.4.4. Asymptotic properties of L_n and M_n 79
4.4.5. Arcsine law 83
4.4.6. Large deviations for random walks 84
4.5. Notes 90

Chapter 5. Critical BPREs: the Annealed Approach 91

5.1. Introduction 91
5.2. Changes of measures 92
5.3. Properties of the prospective minima 99
5.4. Survival probability 107
5.5. Limit theorems for the critical case (annealed approach) 116
5.6. Environment providing survival 122
5.7. Convergence of $\log Z_n$ 126
5.8. Notes 127

Chapter 6. Critical BPREs: the Quenched Approach 133

6.1. Introduction 133
6.2. Changes of measures 134
6.3. Probability of survival 138
6.4. Yaglom limit theorems 142
6.4.1. The population size at non-random moments 143
6.4.2. The population size at moments $nt, 0 < t < 1$ 149
6.4.3. The number of particles at moment $\tau(n) \leq nt$ 150
6.4.4. The number of particles at moment $\tau(n) > nt$ 157
6.5. Discrete limit distributions 159
6.6. Notes 167

Chapter 7. Weakly Subcritical BPREs 169

7.1. Introduction 169
7.2. The probability measures $\widehat{\mathbb{P}}^+$ and $\widehat{\mathbb{P}}^-$ 172
7.3. Proof of theorems 180
7.3.1. Proof of Theorem 7.1 188

7.3.2. Proof of Theorem 7.2 190
7.3.3. Proof of Theorem 7.3 191
7.4. Notes 195

Chapter 8. Intermediate Subcritical BPREs 197

8.1. Introduction 197
8.2. Proof of Theorems 8.1 to 8.3 201
8.3. Further limit results 209
8.4. Conditioned family trees 212
8.5. Proof of Theorem 8.4 219
8.6. Notes 225

Chapter 9. Strongly Subcritical BPREs 227

9.1. Introduction 227
9.2. Survival probability and Yaglom-type limit theorems 229
9.3. Environments providing survival and dynamics of the population size 234
9.3.1. Properties of the transition matrix P^* 240
9.3.2. Proof of Theorem 9.2 243
9.3.3. Proof of Theorem 9.3 248
9.4. Notes 251

Chapter 10. Multi-type BPREs 257

10.1. Introduction 257
10.2. Supercritical MBPREs 264
10.3. The survival probability of subcritical and critical MBPREs 264
10.4. Functional limit theorem in the critical case 266
10.5. Subcritical multi-type case 266

Appendix 269

Bibliography 275

Index 285