

# Table of Contents

<b>Introduction . . . . .</b>	11
<b>Chapter 1 Single-Component Systems . . . . .</b>	17
1.1 Distribution of failure and reliability . . . . .	17
1.1.1 Function of distribution and density of failure . . . . .	17
1.1.2 Survival function: reliability . . . . .	18
1.1.3 Hazard rate . . . . .	19
1.1.4 Maintainability . . . . .	19
1.1.5 Mean times . . . . .	20
1.1.6 Mean residual lifetime . . . . .	21
1.1.7 Fundamental relationships . . . . .	21
1.1.8 Some probability distributions . . . . .	22
1.2 Availability of the repairable systems . . . . .	25
1.2.1 Instantaneous availability . . . . .	25
1.2.2 Asymptotic availability . . . . .	26
1.2.3 Mean availability . . . . .	26
1.2.4 Asymptotic mean availability . . . . .	27
1.3 Reliability in discrete time . . . . .	27
1.3.1 Discrete distributions . . . . .	28
1.3.2 Reliability . . . . .	28
1.4 Reliability and maintenance . . . . .	29
1.4.1 Periodic test: repair time is negligible . . . . .	29
1.4.2 Periodic test: repair time is not negligible . . . . .	30
1.4.3 Mean duration of a hidden failure . . . . .	30
1.5 Reliability data . . . . .	31
<b>Chapter 2 Multi-Component Systems . . . . .</b>	33
2.1 Structure function . . . . .	33

## 6 Fault Trees

2.2	Modules and modular decomposition . . . . .	36
2.3	Elementary structure systems . . . . .	37
2.3.1	Series system . . . . .	37
2.3.2	Parallel system . . . . .	38
2.3.3	System $k$ -out-of- $n$ . . . . .	38
2.3.4	Parallel-series system . . . . .	39
2.3.5	Series-parallel system . . . . .	39
2.4	Systems with complex structure . . . . .	40
2.5	Probabilistic study of the systems . . . . .	42
2.5.1	Introduction . . . . .	42
2.5.2	Inclusion-exclusion method . . . . .	43
2.5.3	Disjoint products . . . . .	44
2.5.4	Factorization . . . . .	46
2.5.5	Reliability bounds . . . . .	46
<b>Chapter 3 Construction of Fault Trees . . . . .</b>		49
3.1	Basic ideas and definitions . . . . .	49
3.1.1	Graphic symbols . . . . .	52
3.1.2	Use of the operators . . . . .	53
3.2	Formal definition and graphs . . . . .	56
3.3	Stages of construction . . . . .	57
3.3.1	Preliminary analysis . . . . .	58
3.3.2	Specifications . . . . .	59
3.3.3	Construction . . . . .	59
3.4	Example of construction . . . . .	60
3.4.1	Preliminary analysis . . . . .	60
3.4.2	Specifications . . . . .	62
3.4.3	Construction . . . . .	62
3.5	Automatic construction . . . . .	63
<b>Chapter 4 Minimal Sets . . . . .</b>		67
4.1	Introduction . . . . .	67
4.2	Methods of study . . . . .	68
4.2.1	Direct methods . . . . .	68
4.2.2	Descending methods . . . . .	71
4.2.3	Ascending methods . . . . .	73
4.3	Reduction . . . . .	74
4.4	Other algorithms for searching the cut sets . . . . .	75
4.5	Inversion of minimal cut sets . . . . .	76
4.6	Complexity of the search for minimal cut sets . . . . .	78

<b>Chapter 5 Probabilistic Assessment . . . . .</b>	79
5.1 The problem of assessment . . . . .	79
5.2 Direct methods . . . . .	80
5.2.1 AND operator . . . . .	81
5.2.2 OR operator . . . . .	81
5.2.3 Exclusive OR operator . . . . .	82
5.2.4 $k$ -out-of- $n$ operator . . . . .	83
5.2.5 Priority-AND operator . . . . .	83
5.2.6 IF operator . . . . .	83
5.3 Methods of minimal sets . . . . .	84
5.3.1 Inclusion-exclusion development . . . . .	84
5.3.2 Disjoint products . . . . .	85
5.3.3 Kitt method . . . . .	86
5.4 Method of factorization . . . . .	88
5.5 Direct recursive methods . . . . .	90
5.5.1 Recursive inclusion-exclusion method . . . . .	90
5.5.2 Method of recursive disjoint products . . . . .	91
5.6 Other methods for calculating the fault trees . . . . .	92
5.7 Large fault trees . . . . .	93
5.7.1 Method of Modarres and Dezfuli [MOD 84] . . . . .	93
5.7.2 Method of Hughes [HUG 87] . . . . .	94
5.7.3 Schneeweiss method [SCH 87] . . . . .	95
5.7.4 Brown method [BRO 90] . . . . .	95
<b>Chapter 6 Influence Assessment . . . . .</b>	97
6.1 Uncertainty . . . . .	97
6.1.1 Introduction . . . . .	97
6.1.2 Methods for evaluating the uncertainty . . . . .	98
6.1.3 Evaluation of the moments . . . . .	99
6.2 Importance . . . . .	103
6.2.1 Introduction . . . . .	103
6.2.2 Structural importance factors . . . . .	105
6.2.3 Probabilistic importance factors . . . . .	106
6.2.4 Importance factors over the uncertainty . . . . .	109
<b>Chapter 7 Modules – Phases – Common Modes . . . . .</b>	111
7.1 Introduction . . . . .	111
7.2 Modular decomposition of an FT . . . . .	111
7.2.1 Module and better modular representation . . . . .	111
7.2.2 Modularization of a fault tree . . . . .	114
7.3 Multiphase fault trees . . . . .	116
7.3.1 Example . . . . .	117

## 8 Fault Trees

7.3.2 Transformation of a multiphase system . . . . .	118
7.3.3 Method of eliminating the minimal cut sets . . . . .	118
7.4 Common mode failures . . . . .	119

## Chapter 8 Extensions: Non-Coherent, Delay and Multistate Fault Trees . . . . .

8.1 Non-coherent fault trees . . . . .	123
8.1.1 Introduction . . . . .	123
8.1.2 An example of a non-coherent FT . . . . .	126
8.1.3 Prime implicants and implicants . . . . .	126
8.1.4 Probabilistic study . . . . .	128
8.2 Delay fault trees . . . . .	129
8.2.1 Introduction . . . . .	129
8.2.2 Treatment . . . . .	129
8.3 FTs and multistate systems . . . . .	131
8.3.1 Multistate systems . . . . .	131
8.3.2 Structure function . . . . .	132
8.3.3 Stochastic description and function of reliability . . . . .	135
8.3.4 Fault trees with restrictions . . . . .	136
8.3.5 Multistate fault trees . . . . .	138

## Chapter 9 Binary Decision Diagrams . . . . .

9.1 Introduction . . . . .	143
9.2 Reduction of the Shannon tree . . . . .	143
9.2.1 Graphical representation of a BDD . . . . .	143
9.2.2 Formal BDD . . . . .	145
9.2.3 Probabilistic calculation . . . . .	147
9.3 Probabilistic assessment of the FTs based on the BDD . . . . .	148
9.4 Research about the prime implicants . . . . .	151
9.5 Algorithmic complexity . . . . .	153

## Chapter 10 Stochastic Simulation of Fault Trees . . . . .

10.1 Introduction . . . . .	155
10.2 Generation of random variables . . . . .	155
10.2.1 Generation of a uniform variable . . . . .	155
10.2.2 Generation of discrete random variables . . . . .	157
10.2.3 Generation of real random variables . . . . .	158
10.3 Implementation and evaluation of the method . . . . .	159
10.3.1 The Monte Carlo method . . . . .	159
10.3.2 Estimating the probability of the top event . . . . .	160

10.3.3	Precision of the estimation . . . . .	161
10.3.4	Acceleration of the convergence . . . . .	164
10.3.5	Rare events . . . . .	165
<b>Exercises</b>	. . . . .	167
<b>Appendices</b>	. . . . .	177
A	BDD Algorithms in FT Analysis . . . . .	179
A1	Introduction . . . . .	179
A2	Obtaining the BDD . . . . .	180
A3	Algorithm of probabilistic assessment . . . . .	182
A4	Importance factors . . . . .	183
A5	Prime implicants . . . . .	184
B	European Benchmark Fault Trees . . . . .	187
B1	Description of the data . . . . .	187
B2	Fault tree: Europe-1 . . . . .	188
B2.1	Structure of the fault tree (structural data) . . . . .	188
B2.2	Probabilistic data . . . . .	190
B2.3	Results . . . . .	190
B3	Fault tree: Europe-2 . . . . .	191
B3.1	Structure of the fault tree . . . . .	191
B3.2	Probabilistic data . . . . .	192
B3.3	Results . . . . .	192
B4	Fault tree: Europe-3 . . . . .	193
B4.1	Structure of the FT . . . . .	193
B4.2	Probabilistic data . . . . .	195
B4.3	Results . . . . .	195
C	Some Results of Probabilities . . . . .	197
<b>Main Notations</b>	. . . . .	201
<b>Bibliography</b>	. . . . .	205
<b>Index</b>	. . . . .	221