

## Table of Contents

<b>Preface . . . . .</b>	13
<b>Chapter 1. Equations of Motion in Non-dissipative Fluid . . . . .</b>	15
1.1. Introduction . . . . .	15
1.1.1. Basic elements . . . . .	15
1.1.2. Mechanisms of transmission . . . . .	16
1.1.3. Acoustic motion and driving motion . . . . .	17
1.1.4. Notion of frequency . . . . .	17
1.1.5. Acoustic amplitude and intensity. . . . .	18
1.1.6. Viscous and thermal phenomena. . . . .	19
1.2. Fundamental laws of propagation in non-dissipative fluids . . . . .	20
1.2.1. Basis of thermodynamics . . . . .	20
1.2.2. Lagrangian and Eulerian descriptions of fluid motion . . . . .	25
1.2.3. Expression of the fluid compressibility: mass conservation law . . . . .	27
1.2.4. Expression of the fundamental law of dynamics: Euler's equation. . . . .	29
1.2.5. Law of fluid behavior: law of conservation of thermomechanic energy . . . . .	30
1.2.6. Summary of the fundamental laws. . . . .	31
1.2.7. Equation of equilibrium of moments . . . . .	32
1.3. Equation of acoustic propagation . . . . .	33
1.3.1. Equation of propagation . . . . .	33
1.3.2. Linear acoustic approximation . . . . .	34
1.3.3. Velocity potential. . . . .	38
1.3.4. Problems at the boundaries . . . . .	40
1.4. Density of energy and energy flow, energy conservation law . . . . .	42
1.4.1. Complex representation in the Fourier domain . . . . .	42
1.4.2. Energy density in an “ideal” fluid . . . . .	43
1.4.3. Energy flow and acoustic intensity . . . . .	45
1.4.4. Energy conservation law. . . . .	48

## 6 Fundamentals of Acoustics

<b>Chapter 1: Appendix. Some General Comments on Thermodynamics . . . . .</b>	50
A.1. Thermodynamic equilibrium and equation of state . . . . .	50
A.2. Digression on functions of multiple variables (study case of two variables) . . . . .	51
A.2.1. Implicit functions . . . . .	51
A.2.2. Total exact differential form . . . . .	53
<b>Chapter 2. Equations of Motion in Dissipative Fluid . . . . .</b>	55
2.1. Introduction . . . . .	55
2.2. Propagation in viscous fluid: Navier-Stokes equation . . . . .	56
2.2.1. Deformation and strain tensor . . . . .	57
2.2.2. Stress tensor . . . . .	62
2.2.3. Expression of the fundamental law of dynamics . . . . .	64
2.3. Heat propagation: Fourier equation . . . . .	70
2.4. Molecular thermal relaxation . . . . .	72
2.4.1. Nature of the phenomenon . . . . .	72
2.4.2. Internal energy, energy of translation, of rotation and of vibration of molecules . . . . .	74
2.4.3. Molecular relaxation: delay of molecular vibrations . . . . .	75
2.5. Problems of linear acoustics in dissipative fluid at rest . . . . .	77
2.5.1. Propagation equations in linear acoustics . . . . .	77
2.5.2. Approach to determine the solutions . . . . .	81
2.5.3. Approach of the solutions in presence of acoustic sources . . . . .	84
2.5.4. Boundary conditions . . . . .	85
<b>Chapter 2: Appendix. Equations of continuity and equations at the thermomechanic discontinuities in continuous media . . . . .</b>	93
A.1. Introduction . . . . .	93
A.1.1. Material derivative of volume integrals . . . . .	93
A.1.2. Generalization . . . . .	96
A.2. Equations of continuity . . . . .	97
A.2.1. Mass conservation equation . . . . .	97
A.2.2. Equation of impulse continuity . . . . .	98
A.2.3. Equation of entropy continuity . . . . .	99
A.2.4. Equation of energy continuity . . . . .	99
A.3. Equations at discontinuities in mechanics . . . . .	102
A.3.1. Introduction . . . . .	102
A.3.2. Application to the equation of impulse conservation . . . . .	103
A.3.3. Other conditions at discontinuities . . . . .	106
A.4. Examples of application of the equations at discontinuities in mechanics: interface conditions . . . . .	106
A.4.1. Interface solid – viscous fluid . . . . .	107
A.4.2. Interface between perfect fluids . . . . .	108
A.4.3 Interface between two non-miscible fluids in motion . . . . .	109

<b>Chapter 3. Problems of Acoustics in Dissipative Fluids . . . . .</b>	111
3.1. Introduction . . . . .	111
3.2. Reflection of a harmonic wave from a rigid plane. . . . .	111
3.2.1. Reflection of an incident harmonic plane wave . . . . .	111
3.2.2. Reflection of a harmonic acoustic wave . . . . .	115
3.3. Spherical wave in infinite space: Green's function . . . . .	118
3.3.1. Impulse spherical source. . . . .	118
3.3.2. Green's function in three-dimensional space. . . . .	121
3.4. Digression on two- and one-dimensional Green's functions in non-dissipative fluids . . . . .	125
3.4.1. Two-dimensional Green's function . . . . .	125
3.4.2. One-dimensional Green's function . . . . .	128
3.5. Acoustic field in "small cavities" in harmonic regime . . . . .	131
3.6. Harmonic motion of a fluid layer between a vibrating membrane and a rigid plate, application to the capillary slit . . . . .	136
3.7. Harmonic plane wave propagation in cylindrical tubes: propagation constants in "large" and "capillary" tubes . . . . .	141
3.8. Guided plane wave in dissipative fluid. . . . .	148
3.9. Cylindrical waveguide, system of distributed constants. . . . .	151
3.10. Introduction to the thermoacoustic engines (on the use of phenomena occurring in thermal boundary layers) . . . . .	154
3.11. Introduction to acoustic gyrometry (on the use of the phenomena occurring in viscous boundary layers) . . . . .	162
<b>Chapter 4. Basic Solutions to the Equations of Linear Propagation in Cartesian Coordinates . . . . .</b>	169
4.1. Introduction . . . . .	169
4.2. General solutions to the wave equation . . . . .	173
4.2.1. Solutions for propagative waves . . . . .	173
4.2.2. Solutions with separable variables . . . . .	176
4.3. Reflection of acoustic waves on a locally reacting surface . . . . .	178
4.3.1. Reflection of a harmonic plane wave . . . . .	178
4.3.2. Reflection from a locally reacting surface in random incidence . . . . .	183
4.3.3. Reflection of a harmonic spherical wave from a locally reacting plane surface . . . . .	184
4.3.4. Acoustic field before a plane surface of impedance Z under the load of a harmonic plane wave in normal incidence . . . . .	185
4.4. Reflection and transmission at the interface between two different fluids . . . . .	187
4.4.1. Governing equations . . . . .	187
4.4.2. The solutions . . . . .	189
4.4.3. Solutions in harmonic regime. . . . .	190
4.4.4. The energy flux . . . . .	192

## 8 Fundamentals of Acoustics

4.5. Harmonic waves propagation in an infinite waveguide with rectangular cross-section. . . . .	193
4.5.1. The governing equations. . . . .	193
4.5.2. The solutions . . . . .	195
4.5.3. Propagating and evanescent waves . . . . .	197
4.5.4. Guided propagation in non-dissipative fluid . . . . .	200
4.6. Problems of discontinuity in waveguides . . . . .	206
4.6.1. Modal theory . . . . .	206
4.6.2. Plane wave fields in waveguide with section discontinuities . . . . .	207
4.7. Propagation in horns in non-dissipative fluids . . . . .	210
4.7.1. Equation of horns. . . . .	210
4.7.2. Solutions for infinite exponential horns. . . . .	214
<b>Chapter 4: Appendix. Eigenvalue Problems, Hilbert Space . . . . .</b>	<b>217</b>
A.1. Eigenvalue problems . . . . .	217
A.1.1. Properties of eigenfunctions and associated eigenvalues . . . . .	217
A.1.2. Eigenvalue problems in acoustics . . . . .	220
A.1.3. Degeneracy . . . . .	220
A.2. Hilbert space . . . . .	221
A.2.1. Hilbert functions and $\mathcal{L}^2$ space . . . . .	221
A.2.2. Properties of Hilbert functions and complete discrete ortho-normal basis . . . . .	222
A.2.3. Continuous complete ortho-normal basis . . . . .	223
<b>Chapter 5. Basic Solutions to the Equations of Linear Propagation in Cylindrical and Spherical Coordinates . . . . .</b>	<b>227</b>
5.1. Basic solutions to the equations of linear propagation in cylindrical coordinates . . . . .	227
5.1.1. General solution to the wave equation . . . . .	227
5.1.2. Progressive cylindrical waves: radiation from an infinitely long cylinder in harmonic regime . . . . .	231
5.1.3. Diffraction of a plane wave by a cylinder characterized by a surface impedance. . . . .	236
5.1.4. Propagation of harmonic waves in cylindrical waveguides . . . . .	238
5.2. Basic solutions to the equations of linear propagation in spherical coordinates . . . . .	245
5.2.1. General solution of the wave equation . . . . .	245
5.2.2. Progressive spherical waves . . . . .	250
5.2.3. Diffraction of a plane wave by a rigid sphere . . . . .	258
5.2.4. The spherical cavity . . . . .	262
5.2.5. Digression on monopolar, dipolar and 2n-polar acoustic fields . . . . .	266

<b>Chapter 6. Integral Formalism in Linear Acoustics . . . . .</b>	277
6.1. Considered problems . . . . .	277
6.1.1. Problems . . . . .	277
6.1.2. Associated eigenvalues problem . . . . .	278
6.1.3. Elementary problem: Green's function in infinite space . . . . .	279
6.1.4. Green's function in finite space . . . . .	280
6.1.5. Reciprocity of the Green's function . . . . .	294
6.2. Integral formalism of boundary problems in linear acoustics . . . . .	296
6.2.1. Introduction . . . . .	296
6.2.2. Integral formalism . . . . .	297
6.2.3. On solving integral equations . . . . .	300
6.3. Examples of application . . . . .	309
6.3.1. Examples of application in the time domain . . . . .	309
6.3.2. Examples of application in the frequency domain . . . . .	318
<b>Chapter 7. Diffusion, Diffraction and Geometrical Approximation . . . . .</b>	357
7.1. Acoustic diffusion: examples . . . . .	357
7.1.1. Propagation in non-homogeneous media . . . . .	357
7.1.2. Diffusion on surface irregularities . . . . .	360
7.2. Acoustic diffraction by a screen . . . . .	362
7.2.1. Kirchhoff-Fresnel diffraction theory . . . . .	362
7.2.2. Fraunhofer's approximation . . . . .	364
7.2.3. Fresnel's approximation . . . . .	366
7.2.4. Fresnel's diffraction by a straight edge . . . . .	369
7.2.5. Diffraction of a plane wave by a semi-infinite rigid plane: introduction to Sommerfeld's theory . . . . .	371
7.2.6. Integral formalism for the problem of diffraction by a semi-infinite plane screen with a straight edge . . . . .	376
7.2.7. Geometric Theory of Diffraction of Keller (GTD) . . . . .	379
7.3. Acoustic propagation in non-homogeneous and non-dissipative media in motion, varying "slowly" in time and space: geometric approximation . . . . .	385
7.3.1. Introduction . . . . .	385
7.3.2. Fundamental equations . . . . .	386
7.3.3. Modes of perturbation . . . . .	388
7.3.4. Equations of rays . . . . .	392
7.3.5. Applications to simple cases . . . . .	397
7.3.6. Fermat's principle . . . . .	403
7.3.7. Equation of parabolic waves . . . . .	405
<b>Chapter 8. Introduction to Sound Radiation and Transparency of Walls . . . . .</b>	409
8.1. Waves in membranes and plates . . . . .	409
8.1.1. Longitudinal and quasi-longitudinal waves . . . . .	410
8.1.2. Transverse shear waves . . . . .	412

## 10 Fundamentals of Acoustics

8.1.3. Flexural waves . . . . .	413
8.2. Governing equation for thin, plane, homogeneous and isotropic plate in transverse motion . . . . .	419
8.2.1. Equation of motion of membranes . . . . .	419
8.2.2. Thin, homogeneous and isotropic plates in pure bending . . . . .	420
8.2.3. Governing equations of thin plane walls . . . . .	424
8.3. Transparency of infinite thin, homogeneous and isotropic walls . . . . .	426
8.3.1. Transparency to an incident plane wave . . . . .	426
8.3.2. Digressions on the influence and nature of the acoustic field on both sides of the wall . . . . .	431
8.3.3. Transparency of a multilayered system: the double leaf system . . . . .	434
8.4. Transparency of finite thin, plane and homogeneous walls: modal theory . . . . .	438
8.4.1. Generally . . . . .	438
8.4.2. Modal theory of the transparency of finite plane walls . . . . .	439
8.4.3. Applications: rectangular plate and circular membrane . . . . .	444
8.5. Transparency of infinite thick, homogeneous and isotropic plates . . . . .	450
8.5.1. Introduction . . . . .	450
8.5.2. Reflection and transmission of waves at the interface fluid-solid . . . . .	450
8.5.3. Transparency of an infinite thick plate . . . . .	457
8.6. Complements in vibro-acoustics: the Statistical Energy Analysis (SEA) method . . . . .	461
8.6.1. Introduction . . . . .	461
8.6.2. The method . . . . .	461
8.6.3. Justifying approach . . . . .	463
<b>Chapter 9. Acoustics in Closed Spaces . . . . .</b>	<b>465</b>
9.1. Introduction . . . . .	465
9.2. Physics of acoustics in closed spaces: modal theory . . . . .	466
9.2.1. Introduction . . . . .	466
9.2.2. The problem of acoustics in closed spaces . . . . .	468
9.2.3. Expression of the acoustic pressure field in closed spaces . . . . .	471
9.2.4. Examples of problems and solutions . . . . .	477
9.3. Problems with high modal density: statistically quasi-uniform acoustic fields . . . . .	483
9.3.1. Distribution of the resonance frequencies of a rectangular cavity with perfectly rigid walls . . . . .	483
9.3.2. Steady state sound field at “high” frequencies . . . . .	487
9.3.3. Acoustic field in transient regime at high frequencies . . . . .	494
9.4. Statistical analysis of diffused fields . . . . .	497
9.4.1. Characteristics of a diffused field . . . . .	497
9.4.2. Energy conservation law in rooms . . . . .	498
9.4.3. Steady-state radiation from a punctual source . . . . .	500
9.4.4. Other expressions of the reverberation time . . . . .	502

9.4.5. Diffused sound fields. . . . .	504
9.5. Brief history of room acoustics . . . . .	508
<b>Chapter 10. Introduction to Non-linear Acoustics, Acoustics in Uniform Flow, and Aero-acoustics. . . . .</b>	<b>511</b>
10.1. Introduction to non-linear acoustics in fluids initially at rest . . . . .	511
10.1.1. Introduction . . . . .	511
10.1.2. Equations of non-linear acoustics: linearization method . . . . .	513
10.1.3. Equations of propagation in non-dissipative fluids in one dimension, Fubini's solution of the implicit equations. . . . .	529
10.1.4. Bürger's equation for plane waves in dissipative (visco-thermal) media . . . . .	536
10.2. Introduction to acoustics in fluids in subsonic uniform flows . . . . .	547
10.2.1. Doppler effect . . . . .	547
10.2.2. Equations of motion . . . . .	549
10.2.3. Integral equations of motion and Green's function in a uniform and constant flow . . . . .	551
10.2.4. Phase velocity and group velocity, energy transfer – case of the rigid-walled guides with constant cross-section in uniform flow . . . . .	556
10.2.5. Equation of dispersion and propagation modes: case of the rigid-walled guides with constant cross-section in uniform flow . . . . .	560
10.2.6. Reflection and refraction at the interface between two media in relative motion (at subsonic velocity) . . . . .	562
10.3. Introduction to aero-acoustics . . . . .	566
10.3.1. Introduction . . . . .	566
10.3.2. Reminder about linear equations of motion and fundamental sources . . . . .	566
10.3.3. Lighthill's equation . . . . .	568
10.3.4. Solutions to Lighthill's equation in media limited by rigid obstacles: Curle's solution . . . . .	570
10.3.5. Estimation of the acoustic power of quadrupolar turbulences . . . . .	574
10.3.6. Conclusion . . . . .	574
<b>Chapter 11. Methods in Electro-acoustics . . . . .</b>	<b>577</b>
11.1. Introduction . . . . .	577
11.2. The different types of conversion . . . . .	578
11.2.1. Electromagnetic conversion . . . . .	578
11.2.2. Piezoelectric conversion (example) . . . . .	583
11.2.3. Electrodynamic conversion . . . . .	588
11.2.4. Electrostatic conversion . . . . .	589
11.2.5. Other conversion techniques . . . . .	591
11.3. The linear mechanical systems with localized constants . . . . .	592
11.3.1. Fundamental elements and systems . . . . .	592
11.3.2. Electromechanical analogies . . . . .	596

## 12 Fundamentals of Acoustics

11.3.3. Digression on the one-dimensional mechanical systems with distributed constants: longitudinal motion of a beam. . . . .	601
11.4. Linear acoustic systems with localized and distributed constants . . . . .	604
11.4.1. Linear acoustic systems with localized constants . . . . .	604
11.4.2. Linear acoustic systems with distributed constants: the cylindrical waveguide . . . . .	611
11.5. Examples of application to electro-acoustic transducers. . . . .	613
11.5.1. Electrodynamic transducer. . . . .	613
11.5.2. The electrostatic microphone . . . . .	619
11.5.3. Example of piezoelectric transducer . . . . .	624
<b>Chapter 11: Appendix</b> . . . . .	626
A.1 Reminder about linear electrical circuits with localized constants. . . . .	626
A.2 Generalization of the coupling equations . . . . .	628
<b>Bibliography</b> . . . . .	631
<b>Index</b> . . . . .	633