Contents

Preface ix

Chapter 1. Presentation of the Formal Computation of Factorization 1

1.1. Definition of the model problem and its functional framework 1
1.2. Direct invariant embedding 4
1.3. Backward invariant embedding 12
1.4. Internal invariant embedding 18

Chapter 2. Justification of the Factorization Computation 23

2.1. Functional framework 23
2.2. Semi-discretization 26
2.3. Passing to the limit 31

Chapter 3. Complements to the Model Problem 41

3.1. An alternative method for obtaining the factorization 41
3.2. Other boundary conditions 43
3.2.1. Boundary conditions on the lateral boundary Σ 43
3.2.2. Boundary conditions on the faces Γ_0 and Γ_a 48
3.2.3. Robin-to-Neumann operator 49
3.2.4. Neumann problem 51

3.3. Explicitly taking into account the boundary conditions and the right-hand side 52
3.4. Periodic boundary conditions in x 58
3.5. An alternative but unstable formulation 59
3.6. Link with the Steklov–Poincaré operator 61
3.7. Application of the Schwarz kernel theorem; link with Green's functions and Hadamard's formula 63

Chapter 4. Interpretation of the Factorization through a Control Problem 69

4.1. Formulation of problem $(\mathcal{P}_0)$ in terms of optimal control 69
4.2. Summary of results on the decoupling of optimal control problems 73
4.3. Summary of results of A. Bensoussan on Kalman optimal filtering 77
4.4. Parabolic regularization for the factorization of elliptic boundary value problems 78
4.4.1. Convergence of the operator P_ε 84
4.4.2. Parabolic regularization for the Neumann-to-Dirichlet operator 94

Chapter 5. Factorization of the Discretized Problem 99

5.1. Introduction and problem statement 99
5.2. Application of factorization method to problem $(\mathcal{P}_h)$ 102
5.3. A second method of discretization 108
5.4. A third possibility: centered scheme 111
5.5. Row permutation 114
5.6. Case of a discretization of the section by finite elements 118

Chapter 6. Other Problems 127

6.1. General second-order linear elliptic problems 127
6.1.1. Problem statement 127
6.1.2. Factorization by invariant embedding 128
6.2. Systems of coupled boundary value problems 133

6.2.1. Global approach 134
6.2.2. Sequential approach 135
6.3. Linear elasticity system 141
6.3.1. Problem statement and transformation 141
6.3.2. Derivation of the decoupled system 145
6.3.3. Associated control problem 147
6.4. Problems of order higher than 2 149
6.4.1. A factorization of the bilaplacian 149
6.4.2. Another (unstable) factorization of the bilaplacian 152
6.5. Stokes problems 155
6.6. Parabolic problems 163

Chapter 7. Other Shapes of Domain 169

7.1. Domain generalization: transformation preserving orthogonal coordinates 169
7.1.1. Hypotheses on the domain 170
7.1.2. Formal derivation 172
7.2. Quasi-cylindrical domains with normal velocity fields 176
7.3. Sweeping the domain by surfaces of arbitrary shape 181

Chapter 8. Factorization by the QR Method 199

8.1. Normal equation for problem $(\mathcal{P}_0)$ in section 1.1 199
8.2. Factorization of the normal equation by invariant embedding 201
8.3. The QR method 207

Chapter 9. Representation Formulas for Solutions of Riccati Equations 213

9.1. Representation formulas 213
9.2. Diagonalization of the two-point boundary value problem 215
9.3. Homographic representation of $P(x)$ 217
9.4. Factorization of problem $(\mathcal{P}_0)$ with a Dirichlet condition at $x = 0$ 220

Appendix 221

Bibliography 233

Index 237