
Contents

Preface	ix
Introduction	xi
Part 1. Piecewise Deterministic Markov Processes and Quantization	1
Chapter 1. Piecewise Deterministic Markov Processes	3
1.1. Introduction	3
1.2. Notation	4
1.3. Definition of a PDMP	5
1.4. Regularity assumptions	8
1.4.1. Lipschitz continuity along the flow	8
1.4.2. Regularity assumptions on the local characteristics	9
1.5. Time-augmented process	11
1.6. Embedded Markov chain	15
1.7. Stopping times	16
1.8. Examples of PDMPs	20
1.8.1. Poisson process with trend	20
1.8.2. TCP	21
1.8.3. Air conditioning unit	22
1.8.4. Crack propagation model	23
1.8.5. Repair workshop model	24
Chapter 2. Examples in Reliability	27
2.1. Introduction	27
2.2. Structure subject to corrosion	28

2.2.1. PDMP model	29
2.2.2. Deterministic time to reach the boundary	32
2.3. The heated hold-up tank	33
2.3.1. Tank dynamics	34
2.3.2. PDMP model	36
Chapter 3. Quantization Technique	39
3.1. Introduction	39
3.2. Optimal quantization	40
3.2.1. Optimal quantization of a random variable	40
3.2.2. Optimal quantization of a Markov chain	42
3.3. Simulation of PDMPs	44
3.3.1. Simulation of time-dependent intensity	45
3.3.2. Simulation of trajectories	45
3.4. Quantization of PDMPs	47
3.4.1. Scale of coordinates of the state variable	48
3.4.2. Cardinality of the mode variable	50
Part 2. Simulation of Functionals	53
Chapter 4. Expectation of Functionals	55
4.1. Introduction	55
4.2. Recursive formulation	57
4.2.1. Lipschitz continuity	58
4.2.2. Iterated operator	60
4.2.3. Approximation scheme	61
4.3. Lipschitz regularity	62
4.4. Rate of convergence	69
4.5. Time-dependent functionals	71
4.6. Deterministic time horizon	74
4.6.1. Direct estimation of the running cost term	74
4.6.2. Bounds of the boundary jump cost term	77
4.6.3. Bounds in the general case	79
4.7. Example	81
4.8. Conclusion	84
Chapter 5. Exit Time	87
5.1. Introduction	87
5.2. Problem setting	88
5.2.1. Distribution	90
5.2.2. Moments	91

5.2.3. Computation horizon	92
5.3. Approximation schemes	92
5.4. Convergence	95
5.4.1. Distribution	95
5.4.2. Moments	100
5.5. Example	101
5.6. Conclusion	108
Chapter 6. Example in Reliability: Service Time	109
6.1. Mean thickness loss	109
6.2. Service time	112
6.2.1. Mean service time	114
6.2.2. Distribution of the service time	118
6.3. Conclusion	121
Part 3. Optimization	123
Chapter 7. Optimal Stopping	125
7.1. Introduction	125
7.2. Dynamic programming equation	128
7.3. Approximation of the value function	130
7.4. Lipschitz continuity properties	132
7.4.1. Lipschitz properties of J and K	132
7.4.2. Lipschitz properties of the value functions	135
7.5. Error estimation for the value function	138
7.5.1. Second term	140
7.5.2. Third term	141
7.5.3. Fourth term	147
7.5.4. Proof of theorem 7.1	148
7.6. Numerical construction of an ϵ -optimal stopping time	149
7.7. Example	161
Chapter 8. Partially Observed Optimal Stopping Problem	165
8.1. Introduction	165
8.2. Problem formulation and assumptions	167
8.3. Optimal filtering	170
8.4. Dynamic programming	175
8.4.1. Preliminary results	176
8.4.2. Optimal stopping problem under complete observation	180
8.4.3. Dynamic programming equation	181
8.5. Numerical approximation by quantization	188

8.5.1. Lipschitz properties	189
8.5.2. Discretization scheme	195
8.5.3. Numerical construction of an ϵ -optimal stopping time	205
8.6. Numerical example	211
Chapter 9. Example in Reliability: Maintenance Optimization	215
9.1. Introduction	215
9.2. Corrosion process	216
9.3. Air conditioning unit	219
9.4. The heated hold-up tank	221
9.4.1. Problem setting and simulation	222
9.4.2. Numerical results and validation	224
9.5. Conclusion	228
Chapter 10. Optimal Impulse Control	231
10.1. Introduction	231
10.2. Impulse control problem	233
10.3. Lipschitz-continuity properties	236
10.3.1. Lipschitz properties of the operators	236
10.3.2. Lipschitz properties of the operator \mathcal{L}	239
10.4. Approximation of the value function	242
10.4.1. Time discretization	245
10.4.2. Approximation of the value functions on the control grid \mathbb{U}	246
10.4.3. Approximation of the value function	255
10.4.4. Step-by-step description of the algorithm	259
10.4.5. Practical implementation	259
10.5. Example	262
10.6. Conclusion	264
Bibliography	269
Index	277