Preface

Contemporary communication systems and computer networks usually have a rather complex structure and therefore require creating more complicated mathematical models of queues and developing new approaches for modeling and asymptotic investigation. The main features of these systems are the stochasticity of the processes describing the behavior in time, influence of various internal and external events which may change (switch) the behavior of the system, the presence of different time scales for different subsystems (very fast internal computer time and user interaction time, etc), and the hierarchic structure. Wide classes of such systems can be adequately described with the help of so-called "switching" stochastic processes.

Switching processes (SP) have been developed by the author for describing the operation of stochastic systems with the property that their development in time varies spontaneously (switches) at some random points of time which may depend on the previous system trajectory. According to Kolmogorov, these processes can be called random processes with discrete interference of chance or with discrete components. Processes of this type often appear in the theory of queueing and communication systems and networks, branching, population and migration processes, in the analysis of stochastic dynamical systems with random perturbations, random movements and various other applications.

SP can be represented as a two-component process $(x(t), \zeta(t))$, $t \geq 0$, with the property that there exists a sequence of Markov points of time $t_1 < t_2 < \cdots$ such that in each interval $[t_k, t_{k+1})$, $x(t) = x(t_k)$, and the behavior of the process $\zeta(t)$ in this interval depends only on the value $(x(t_k), \zeta(t_k))$. $x(t)$ is a discrete switching component and the points of time $\{t_k\}$ are called switching times. SP can be described in terms of constructive characteristics and is very suitable in analyzing and asymptotic investigating of stochastic systems with "rare" and "fast" switching.

The class of SPs is the natural generalization of well-known classes of random processes such as Markov processes that are homogenous in the 2nd component, processes with independent increments and Markov or semi-Markov switches, piecewise

Markov aggregates, and Markov processes with Markov and semi-Markov switching (random evolutions). Wide classes of queueing models can be described in terms of SPs. The class of switching queueing models includes, as examples, various types of state-dependent queueing systems and networks in a Markov or semi-Markov environment, queueing models under the influence of flows of external events or internal perturbations, unreliable systems, retrial queues, hierarchic queueing systems, etc. Therefore, the asymptotic theory of SPs can be effectively applied to the investigation of wide classes of queueing systems and networks.

In the book several large directions of asymptotic results for SP are investigated and successfully applied to various classes of switching queueing models.

The first direction is devoted to the limit theorems of averaging principle (AP) and diffusion approximation (DA) type in the case of fast switching. Theorems on the convergence of the trajectory of an SP to a solution of a differential equation (AP) and the convergence of the normalized difference to a diffusion process (DA) are proved for different subclasses of SP: recurrent processes of a semi-Markov type (RPSMs), processes with semi-Markov switching and general SP with feedback between both components. The results are based on the investigation of the asymptotic properties of a special subclass of SP – RPSMs theorems on the convergence of recurrent sequences with Markov switching to the solutions of stochastic differential equations and the convergence of superpositions of random functions.

This class of theorems is the basis of a new approach to the investigation of transient phenomena for service processes in overloading queueing systems and Markov and semi-Markov type networks, retrial queues, etc. Numerous examples for the illustration AP and DA for queueing models are considered.

The second direction is devoted to the limit theorems for SP with slow switching. Models of this type appear at the investigation of hierarchic systems in different scales of time (slow and fast). The conditions, when an SP of a rather complicated structure can be approximated by an SP of a simpler structure, in particular, by a Markov or semi-Markov process, are established and various applications to processes with Markov and semi-Markov switching are considered. The method of investigation uses the results on the convergence of the accumulating type processes constructed on the trajectory of Markov or semi-Markov process satisfying some form of the asymptotic mixing condition in triangular scheme to processes with independent increments (homogenous or non-homogenous in time). A special class of non-homogenous in time Markov processes with transition probabilities slowly varying in the expanding time scale is introduced. These processes have quasi-ergodic properties and are called quasi-ergodic Markov processes. Under rather general conditions it is proved a Poisson approximation of the flows of rare events governed by a Markov process satisfying an asymptotic mixing condition, in particular with the state space forming a so-called S-set (asymptotically connected set), and the exponential approximation of the exit time from S-set. Special attention is paid to the analysis of the flow of rare events defined on stochastic systems satisfying asymptotic mixing conditions, in particular, with state space forming an S-set. These models naturally appear at study queueing models with asymptotically “fast” service (or low traffic). Applications of a method of S-sets are considered for different classes of queueing systems.

Using these results and the results on the convergence of SP with slow switching, the models of the asymptotic aggregation of the state space of Markov and semi-Markov processes (homogenous and non-homogenous in time) are investigated. These results create the basis for a theory of the asymptotic decreasing dimension and aggregation (consolidation) of the state space of stochastic systems. Special attention is paid to the hierarchic Markov and semi-Markov systems operating in different time scales. These systems under rather general conditions can be approximated by a simpler Markov system with averaged transition characteristics. The applications to the asymptotic aggregation of a state space and approximation by Markov models with averaged characteristics are considered for different classes of Markov and non-Markov queueing models in a random environment.

The asymptotic aggregation of SP in different time scales is the next natural level of development. The conditions of the convergence of SP to solutions of differential and stochastic differential equations with coefficients depending on a limiting aggregated Markov or semi-Markov process are obtained. Various applications to the asymptotic aggregation of overloaded queueing systems and networks under the influence of hierarchic random environment in different time scales are considered.

The results of the book were obtained while the author was working at Kiev University as Head and Professor of Applied Statistics Department at the Faculty of Cybernetics (1978–2002) and also as Visiting Professor at Bilkent University, Ankara (1997–2002). Some results were reflected in different courses on stochastic processes and queueing models that the author taught at Kiev University and Bilkent University for graduate and post-graduate students.

The book contains many practical examples of asymptotic results for queueing models and is directed to applied mathematicians and researchers, post-graduate students and engineers working in the field of stochastic systems, queueing models and applications to computer sciences, biology, ecology, physical and social sciences. Some theoretical results are illustrated by examples of simulation in R.

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