

## Table of Contents

<b>Foreword</b> . . . . .	13
<b>Introduction</b> . . . . .	17
<b>Part I. Particle Swarm Optimization</b> . . . . .	21
<b>Chapter 1. What is a Difficult Problem?</b> . . . . .	23
1.1. An intrinsic definition . . . . .	23
1.2. Estimation and practical measurement . . . . .	25
1.3. For “amateurs”: some estimates of difficulty . . . . .	26
1.3.1. Function $\sum_{d=1}^D x_d$ . . . . .	27
1.3.2. Function $\sum_{d=1}^D x_d^2$ . . . . .	27
1.3.3. Function $\sum_{d=1}^D \sqrt{x_d  \sin(x_d) }$ . . . . .	27
1.3.4. Traveling salesman on D cities . . . . .	28
1.4. Summary . . . . .	28
<b>Chapter 2. On a Table Corner</b> . . . . .	29
2.1. Apiarian metaphor . . . . .	29
2.2. An aside on the spreading of a rumor . . . . .	30
2.3. Abstract formulation . . . . .	30
2.4. What is really transmitted . . . . .	34
2.5. Cooperation <i>versus</i> competition . . . . .	35
2.6. For “amateurs”: a simple calculation of propagation of rumor . . . . .	35
2.7. Summary . . . . .	36

<b>Chapter 3. First Formulations</b> . . . . .	37
3.1. Minimal version . . . . .	37
3.1.1. Swarm size . . . . .	37
3.1.2. Information links . . . . .	38
3.1.3. Initialization . . . . .	38
3.1.4. Equations of motion . . . . .	39
3.1.5. Interval confinement . . . . .	40
3.1.6. Proximity distributions . . . . .	42
3.2. Two common errors . . . . .	44
3.3. Principal drawbacks of this formulation . . . . .	45
3.3.1. Distribution bias . . . . .	45
3.3.2. Explosion and maximum velocity . . . . .	48
3.4. Manual parameter setting . . . . .	48
3.5. For “amateurs”: average number of informants . . . . .	49
3.6. Summary . . . . .	50
<b>Chapter 4. Benchmark Set</b> . . . . .	51
4.1. What is the purpose of test functions? . . . . .	51
4.2. Six reference functions . . . . .	52
4.3. Representations and comments . . . . .	52
4.4. For “amateurs”: estimates of levels of difficulty . . . . .	56
4.4.1. Theoretical difficulty . . . . .	56
4.4.1.1. Tripod . . . . .	56
4.4.1.2. Alpine 10D . . . . .	57
4.4.1.3. Rosenbrock . . . . .	57
4.4.2. Difficulty according to the search effort . . . . .	58
4.5. Summary . . . . .	58
<b>Chapter 5. Mistrusting Chance</b> . . . . .	59
5.1. Analysis of an anomaly . . . . .	59
5.2. Computing randomness . . . . .	61
5.3. Reproducibility . . . . .	61
5.4. On numerical precision . . . . .	62
5.5. The rare KISS . . . . .	62
5.5.1. Brief description . . . . .	63
5.5.2. Test of KISS . . . . .	64
5.6. On the comparison of results . . . . .	64
5.7. For “amateurs”: confidence in the estimate of a rate of failure . . . . .	65
5.8. C programs . . . . .	68
5.9. Summary . . . . .	69

<b>Chapter 6. First Results</b> . . . . .	71
6.1. A simple program . . . . .	71
6.2. Overall results . . . . .	72
6.3. Robustness and performance maps . . . . .	73
6.5. Theoretical difficulty and noted difficulty . . . . .	80
6.6. Source code of OEP 0. . . . .	80
6.7. Summary . . . . .	85
<b>Chapter 7. Swarm: Memory and Graphs of Influence</b> . . . . .	87
7.1. Circular neighborhood of the historical PSO . . . . .	87
7.2. Memory-swarm . . . . .	88
7.3. Fixed topologies . . . . .	90
7.4. Random variable topologies . . . . .	92
7.4.1. Direct recruitment. . . . .	92
7.4.2. Recruitment by common channel of communication . . . . .	92
7.5. Influence of the number of informants . . . . .	93
7.5.1. In fixed topology . . . . .	93
7.5.2. In random variable topology . . . . .	95
7.6. Influence of the number of memories . . . . .	95
7.7. Reorganizations of the memory-swarm . . . . .	97
7.7.1. Mixing of the memories . . . . .	97
7.7.2. Queen and other centroids . . . . .	98
7.7.3. Comparative results. . . . .	98
7.8. For “amateurs”: temporal connectivity in random recruitment. . . . .	99
7.9. Summary . . . . .	101
<b>Chapter 8. Distributions of Proximity</b> . . . . .	103
8.1. The random possibilities . . . . .	103
8.2. Review of rectangular distribution . . . . .	104
8.3. Alternative distributions of possibilities . . . . .	105
8.3.1. Ellipsoidal positive sectors . . . . .	105
8.3.2. Independent Gaussians. . . . .	106
8.3.3. Local by independent Gaussians . . . . .	107
8.3.4. The class of one-dimensional distributions . . . . .	107
8.3.5. Pivots . . . . .	108
8.3.6. Adjusted ellipsoids . . . . .	112
8.4. Some comparisons of results. . . . .	113
8.5. For “amateurs” . . . . .	116
8.5.1. Squaring of a hypersphere . . . . .	116
8.5.2. From sphere to ellipsoid . . . . .	117
8.5.3. Random volume for an adjusted ellipsoid. . . . .	117
8.5.4. Uniform distribution in a D-sphere. . . . .	118
8.6. C program of isotropic distribution . . . . .	118
8.7. Summary . . . . .	119

<b>Chapter 9. Optimal Parameter Settings</b> . . . . .	121
9.1. Defense of manual parameter setting . . . . .	121
9.2. Better parameter settings for the benchmark set . . . . .	122
9.2.1. Search space . . . . .	122
9.2.2. To optimize the optimizer . . . . .	123
9.2.3. Analysis of results . . . . .	125
9.2.3.1. Rate of failure . . . . .	125
9.2.3.2. Distribution . . . . .	125
9.2.3.3. Topology and the number of informants . . . . .	125
9.2.3.4. Informants $K$ . . . . .	125
9.2.3.5. Coefficient $\varphi$ . . . . .	126
9.2.3.6. Informants $N$ and memories $M$ . . . . .	126
9.3. Towards adaptation . . . . .	127
9.4. For “amateurs”: number of graphs of information . . . . .	127
9.5. Summary . . . . .	128
<b>Chapter 10. Adaptations</b> . . . . .	129
10.1. Demanding criteria . . . . .	129
10.1.1. Criterion 1 . . . . .	129
10.1.2. Criterion 2 . . . . .	129
10.2. Rough sketches . . . . .	130
10.2.1. Weighting with temporal decrease . . . . .	130
10.2.2. Selection and replacement . . . . .	131
10.2.3. Parametric adaptations . . . . .	132
10.2.4. Nonparametric adaptations . . . . .	133
10.3. For “amateurs” . . . . .	135
10.3.1. Formulas of temporal decrease . . . . .	135
10.3.2. Parametric adaptations . . . . .	136
10.3.2.1. Case 1 ( $m_i \geq 0$ ) . . . . .	137
10.3.2.2. Case 2 ( $m_i < 0$ ) . . . . .	137
10.4. Summary . . . . .	138
<b>Chapter 11. TRIBES or Cooperatin of Tribes</b> . . . . .	139
11.1. Towards an ultimate program . . . . .	139
11.2. Description of TRIBES . . . . .	141
11.2.1. Tribes . . . . .	141
11.2.2. The tribal relationships . . . . .	141
11.2.3. Quality of a particle . . . . .	141
11.2.4. Quality of a tribe . . . . .	142
11.2.5. Evolution of the tribes . . . . .	142
11.2.5.1. Removal of a particle . . . . .	142
11.2.5.2. Generation of a particle . . . . .	144
11.2.6. Strategies of displacement . . . . .	145

11.2.7. Best informant . . . . .	146
11.2.7.1. Direct comparison, general case . . . . .	147
11.2.7.2. Comparison by pseudo-gradients, metric spaces . . . . .	147
11.3. Results on the benchmark set . . . . .	147
11.4. Summary . . . . .	149
<b>Chapter 12. On the Constraints . . . . .</b>	<b>151</b>
12.1. Some preliminary reflections. . . . .	151
12.2. Representation of the constraints . . . . .	152
12.3. Imperative constraints and indicative constraints . . . . .	153
12.4. Interval confinement. . . . .	154
12.5. Discrete variable . . . . .	154
12.5.1. Direct method . . . . .	155
12.5.1.1. List not ordered (and not orderable) . . . . .	155
12.5.1.2. Ordered list . . . . .	155
12.5.2. Indirect method . . . . .	155
12.6. Granularity confinement . . . . .	156
12.7. “all different” confinement . . . . .	156
12.8. Confinement by dichotomy. . . . .	157
12.9. Multicriterion treatment. . . . .	158
12.10. Treatment by penalties. . . . .	161
12.11. C source code. Dichotomic search in a list. . . . .	162
12.12. For “amateurs” . . . . .	162
12.13. Summary. . . . .	165
<b>Chapter 13. Problems and Application . . . . .</b>	<b>167</b>
13.1. Ecological niche . . . . .	167
13.2. Typology and choice of problems . . . . .	168
13.3. Canonical representation of a problem of optimization . . . . .	169
13.4. Knapsack . . . . .	169
13.5. Magic squares . . . . .	170
13.6. Quadratic assignment . . . . .	171
13.7. Traveling salesman . . . . .	172
13.8. Hybrid JM. . . . .	173
13.9. Training of a neural network. . . . .	174
13.9.1. Exclusive OR. . . . .	175
13.9.2. Diabetes among Pima Indians . . . . .	176
13.9.3. Servomechanism. . . . .	176
13.9.4. Comparisons . . . . .	176
13.10. Pressure vessel . . . . .	177
13.10.1. Continuous relaxed form . . . . .	179
13.10.2. Complete discrete form . . . . .	180
13.11. Compression spring . . . . .	182
13.12. Moving Peaks . . . . .	185

13.13. For “amateurs”: the magic of squares . . . . .	188
13.14. Summary . . . . .	188
<b>chapter 14. Conclusion . . . . .</b>	<b>189</b>
14.1. End of the beginning . . . . .	189
14.2. Mono, poly, meta. . . . .	189
14.3. The beginning of the end? . . . . .	190
<b>Part II. Outlines . . . . .</b>	<b>193</b>
<b>Chapter 15. On Parallelism . . . . .</b>	<b>195</b>
15.1. The short-sighted swarm . . . . .	195
15.2. A parallel model . . . . .	195
15.3. A counter-intuitive result . . . . .	196
15.4. Qualitative explanation . . . . .	197
15.5. For “amateurs”: probability of questioning an improved memory . . .	198
15.6. Summary . . . . .	199
<b>Chapter 16. Combinatorial Problems . . . . .</b>	<b>201</b>
16.1. Difficulty of chaos . . . . .	201
16.2. Like a crystal . . . . .	202
16.3. Confinement method . . . . .	203
16.4. Canonical PSO . . . . .	204
16.5. Summary . . . . .	210
<b>Chapter 17. Dynamics of a Swarm. . . . .</b>	<b>211</b>
17.1. Motivations and tools . . . . .	211
17.2. An example with the magnifying glass . . . . .	212
17.2.1. One particle. . . . .	212
17.2.2. Two particles . . . . .	214
17.3. Energies . . . . .	217
17.3.1. Definitions . . . . .	217
17.3.2. Evolutions . . . . .	218
17.4. For experienced “amateurs”: convergence and constriction . . . . .	220
17.4.1. Criterion of convergence . . . . .	220
17.4.2. Coefficients of constriction . . . . .	221
17.4.3. Positive discriminant . . . . .	222
17.5. Summary . . . . .	224

<b>Chapter 18. Techniques and Alternatives</b> . . . . .	225
18.1. Reprise. . . . .	225
18.2. Stop-restart/reset . . . . .	226
18.2.1. A criterion of abandonment . . . . .	226
18.2.2. Guided re-initialization . . . . .	227
18.3. Multi-swarm . . . . .	227
18.4. Dynamic optimization. . . . .	228
18.5. For “amateurs” . . . . .	229
18.5.1. Maximum flight and criterion of abandonment. . . . .	229
18.5.2. Dilation . . . . .	230
18.6. Summary . . . . .	230
<b>Further Information</b> . . . . .	231
<b>Bibliography</b> . . . . .	233
<b>Index</b> . . . . .	239