

## Table of Contents

<b>Chapter 1. Statistical Properties of Images</b> . . . . .	1
Henri MAÎTRE	
1.1. Introduction . . . . .	1
1.1.1. Why study statistical properties? . . . . .	1
1.1.2. What images? . . . . .	2
1.2. Amplitude . . . . .	3
1.2.1. Properties . . . . .	3
1.2.2. Sensitivity to modifications of histograms . . . . .	4
1.3. Jumps in amplitude . . . . .	7
1.4. The autocorrelation function . . . . .	8
1.4.1. In one dimension . . . . .	8
1.4.2. In multiple dimensions . . . . .	11
1.4.3. The power spectral density . . . . .	12
1.5. Entropy . . . . .	13
1.5.1. Entropy of order 0 . . . . .	14
1.5.2. Entropy of jumps . . . . .	15
1.5.3. Conditional entropy, redundancy . . . . .	15
1.5.4. Return on a paradox . . . . .	16
1.6. Image models . . . . .	17
1.6.1. Markov-Gauss process . . . . .	17
1.6.2. The mosaic model . . . . .	18
1.7. Invariable scaling models . . . . .	19
1.8. Bibliography . . . . .	20
<b>Chapter 2. Image Sampling and Fractal Representation</b> . . . . .	23
Henri MAÎTRE	
2.1. Limited bandwidth one-dimensional signals. . . . .	24
2.1.1. Signal sampling. . . . .	24

2.1.2. Reconstruction of a Nyquist frequency sampled signal . . . . .	25
2.1.3. Reconstruction in the case of oversampling . . . . .	26
2.2. Real signals . . . . .	28
2.2.1. Physical limitations of the spectrum. . . . .	28
2.2.1.1. Optical limits . . . . .	28
2.2.1.2. Solid sensors . . . . .	29
2.2.2. Signal filtration . . . . .	29
2.3. Extension in $N$ dimension. . . . .	31
2.4. The fractal model. . . . .	33
2.4.1. Fractal dimension. . . . .	35
2.4.2. Internal homothety . . . . .	36
2.4.3. What should be done with the fractal theory? . . . . .	36
2.5. Bibliography . . . . .	40
<b>Chapter 3. Discrete Representations</b> . . . . .	41
Isabelle BLOCH	
3.1. Introduction . . . . .	41
3.2. Tessellations and digital grids . . . . .	42
3.2.1. Definitions and constraints . . . . .	42
3.2.2. Regular plane tessellations . . . . .	43
3.2.3. Semi-regular plane tessellations . . . . .	44
3.2.4. Duality between tessellations and grids. . . . .	45
3.3. Discrete topology. . . . .	45
3.3.1. Some approaches . . . . .	46
3.3.2. Topology from the notion of basic neighborhood. . . . .	48
3.3.3. Euler number: an example of a topological characteristic of an object	54
3.4. Geometric representations . . . . .	56
3.4.1. Discretization of a continuous line . . . . .	57
3.4.2. Characterization of a discrete line segment . . . . .	59
3.4.3. Discrete analytical lines . . . . .	61
3.4.4. Discrete circles . . . . .	63
3.4.5. Voronoï tessellations and Delaunay triangulations . . . . .	65
3.5. Examples of random structures . . . . .	68
3.6. Distance function. . . . .	69
3.6.1. Definition of discrete distances . . . . .	70
3.6.2. Examples . . . . .	71
3.6.3. Calculation algorithms. . . . .	71
3.7. Bibliography . . . . .	73
<b>Chapter 4. Image Restoration.</b> . . . . .	75
Henri MAÎTRE	
4.1. Inverse filtering. . . . .	76

4.1.1. Analytical approach . . . . .	76
4.1.2. Algebraic approach . . . . .	77
4.1.3. Discussion . . . . .	78
4.1.4. Conclusions (provisionary) . . . . .	81
4.2. Singular and ill-conditioned defects . . . . .	82
4.2.1. Singular defects . . . . .	82
4.2.2. Ill-conditioned system . . . . .	83
4.2.2.1. Conditioning . . . . .	83
4.2.2.2. The Wiener filter . . . . .	84
4.2.2.3. Estimation of power spectral densities . . . . .	85
4.2.3. Decomposition into singular values . . . . .	86
4.3. Restoration of two-dimensional signals . . . . .	87
4.4. Iterative restoration . . . . .	89
4.4.1. Jacobi method . . . . .	89
4.4.2. Gauss-Seidel method . . . . .	91
4.4.3. Greatest slope method . . . . .	91
4.4.4. Conjugated gradient method . . . . .	91
4.4.5. Projection methods (POCS) . . . . .	92
4.5. Estimating defects . . . . .	92
4.5.1. Determination by calibration . . . . .	93
4.5.2. Statistical determination . . . . .	94
4.6. Reduction of the border effects . . . . .	94
4.7. Bibliography . . . . .	95
<b>Chapter 5. Mathematical Morphology . . . . .</b>	<b>97</b>
Isabelle BLOCH	
5.1. Introduction and preliminaries . . . . .	97
5.2. The four operations . . . . .	100
5.2.1. Structuring element and set theoretical framework . . . . .	100
5.2.2. Binary erosion and dilation . . . . .	101
5.2.3. Erosion and dilation of functions . . . . .	105
5.2.4. Binary openings and closings . . . . .	110
5.2.5. Numerical opening and closing . . . . .	112
5.3. Topological framework . . . . .	113
5.4. Algebraic framework . . . . .	116
5.4.1. Lattices . . . . .	116
5.4.2. Algebraic erosion and dilation, adjunctions . . . . .	117
5.4.3. Link with morphological operators . . . . .	118
5.4.4. Algebraic opening and closing . . . . .	119
5.5. Probabilistic framework . . . . .	119
5.5.1. Random closed sets . . . . .	119
5.5.2. An example: the Boolean scheme . . . . .	120

5.6. Applications of erosion and dilation . . . . .	121
5.6.1. Measures . . . . .	121
5.6.2. Ultimate erosion . . . . .	122
5.6.3. Contrast enhancement . . . . .	122
5.6.4. Morphological gradient . . . . .	123
5.7. Applications of opening and closing . . . . .	124
5.7.1. Sequential alternate filters . . . . .	124
5.7.2. Auto-dual filters . . . . .	125
5.7.3. Top hat . . . . .	125
5.7.4. Granulometries . . . . .	126
5.7.5. Surface opening . . . . .	127
5.7.6. Annular opening . . . . .	127
5.8. Hit-or-miss transformation and derived operators . . . . .	128
5.8.1. Hit-or-miss transformation . . . . .	128
5.8.2. Thinning and thickening . . . . .	128
5.8.3. Skeleton . . . . .	129
5.8.4. Discrete skeleton . . . . .	131
5.8.5. SKIZ . . . . .	132
5.9. Geodesy . . . . .	132
5.9.1. Geodesic distance and geodesic balls . . . . .	132
5.9.2. Geodesic morphological operations, reconstruction . . . . .	133
5.9.3. Geodesic skeleton by influence zones . . . . .	135
5.10. Watershed . . . . .	135
5.10.1. Definition and properties . . . . .	135
5.11. Conclusion . . . . .	139
5.12. Bibliography . . . . .	139
<b>Chapter 6. Markov Fields . . . . .</b>	<b>141</b>
Florence TUPIN and Marc SIGELLE	
6.1. Definition and simulation of a Markov field . . . . .	142
6.1.1. Description of the image . . . . .	142
6.1.2. Probabilistic modeling of the image . . . . .	143
6.1.3. Markov fields – Gibbs fields . . . . .	144
6.1.3.1. Definition of a Markov field . . . . .	144
6.1.3.2. Equivalence between Markov and Gibbs fields . . . . .	144
6.1.4. MRF sampling . . . . .	147
6.1.4.1. Gibbs sampling . . . . .	148
6.1.4.2. The Metropolis algorithm . . . . .	149
6.1.5. Search for the most probable configuration . . . . .	150
6.1.5.1. Gibbs distribution with temperature . . . . .	150
6.1.5.2. Simulated annealing algorithm . . . . .	151
6.1.5.3. Iterated conditional mode (ICM) algorithm . . . . .	153

6.1.6. Some fundamental MRFs . . . . .	154
6.1.6.1. Ising model . . . . .	154
6.1.6.2. Potts model . . . . .	155
6.1.6.3. Markovian Gaussian model . . . . .	156
6.2. Applications: restoration and segmentation . . . . .	157
6.2.1. Bayesian frame . . . . .	157
6.2.2. The case of restoration . . . . .	158
6.2.3. The case of segmentation . . . . .	160
6.3. Markovian frame estimators . . . . .	162
6.3.1. Introduction . . . . .	162
6.3.2. Bayesian modeling and cost function . . . . .	162
6.3.3. MAP estimator . . . . .	163
6.3.4. MPM estimator . . . . .	164
6.3.5. TPM estimator . . . . .	166
6.3.6. Comparison of MAP, MPM and TPM estimators . . . . .	167
6.4. Parameter estimating . . . . .	169
6.4.1. Introduction . . . . .	169
6.4.2. Complete data . . . . .	169
6.4.3. Incomplete data . . . . .	171
6.5. Border process . . . . .	174
6.5.1. Explicit and implicit border process . . . . .	174
6.5.2. Minimization algorithms . . . . .	176
6.6. Primitive graphs . . . . .	177
6.7. Bibliography . . . . .	178
<b>Chapter 7. Wavelets and Image Processing . . . . .</b>	<b>181</b>
Béatrice PESQUET-POPESCU and Jean-Christophe PESQUET	
7.1. Linear image analysis principles . . . . .	181
7.2. Frames . . . . .	187
7.2.1. Objective . . . . .	187
7.2.2. Definition . . . . .	187
7.2.3. Properties . . . . .	188
7.2.4. Dual frame . . . . .	190
7.2.5. Frame algorithm . . . . .	192
7.2.6. Examples of frames . . . . .	194
7.3. Adaptive pursuit . . . . .	198
7.3.1. Motivation . . . . .	198
7.3.2. Matching pursuit algorithm . . . . .	198
7.3.3. Improvements . . . . .	201
7.3.3.1. Reduction of complexity . . . . .	201
7.3.3.2. Orthogonalization . . . . .	202
7.4. Bibliography . . . . .	203

<b>Chapter 8. Partial Derivative Equations and Image Processing</b> . . . . .	205
Yann GOUSSEAU	
8.1. The heat equation and its limitations . . . . .	206
8.1.1. Some notations . . . . .	206
8.1.2. Why the heat equation? . . . . .	207
8.1.3. Heat equation and Gaussian core . . . . .	207
8.1.4. Application in images . . . . .	208
8.1.5. The inverse heat equation. . . . .	209
8.1.6. Limitations, invariance by contrast changes . . . . .	211
8.2. Non-linear diffusion equations. . . . .	213
8.2.1. Notations and local structure of images. . . . .	213
8.2.2. The Malik and Perona equation . . . . .	215
8.2.3. Mean curve and variant movement . . . . .	217
8.2.4. The Rudin-Osher-Fatemi equation . . . . .	219
8.2.5. The Rudin shock filter. . . . .	221
8.3. PDE and multi-scale analysis. . . . .	223
8.3.1. Definition and properties of multi-scale analyses. . . . .	223
8.3.2. Why are multi-scale analyses regulated by PDEs? . . . . .	224
8.3.3. Multi-scale analysis classification . . . . .	226
8.3.3.1. Isotropic linear analysis . . . . .	226
8.3.3.2. Invariance by change of contrast. . . . .	227
8.3.3.3. Affine invariance . . . . .	227
8.4. Bibliography . . . . .	228
<b>Chapter 9. Pre-processing</b> . . . . .	229
Henri MAÎTRE	
9.1. Photometric or colorimetric processes . . . . .	229
9.1.1. Linearity . . . . .	229
9.1.2. Homogeneity . . . . .	230
9.1.3. Binarization, thresholding. . . . .	232
9.1.4. Increase in contrast. . . . .	232
9.2. Noise suppression . . . . .	232
9.2.1. Linear filters. . . . .	232
9.2.1.1. Complexity of linear filters . . . . .	233
9.2.1.2. Wiener filter. . . . .	234
9.2.2. Rank filtering . . . . .	234
9.2.3. Morphologic filters. . . . .	235
9.2.4. Filtering by diffusion equations . . . . .	236
9.2.4.1. Isotropic diffusion . . . . .	236
9.2.4.2. Anisotropic diffusion . . . . .	237
9.2.4.3. Mean curvature filters . . . . .	238
9.2.4.4. Total variation filtering . . . . .	238

9.3. Adaptive filters . . . . .	240
9.3.1. Adaptive coefficient filters . . . . .	240
9.3.1.1. Inverse gradient filter . . . . .	240
9.3.1.2. Saint-Marc filter . . . . .	240
9.3.1.3. Fairfield toboggan . . . . .	240
9.3.2. Adaptive window filters. . . . .	241
9.3.2.1. Nagao filter . . . . .	241
9.3.2.2. Wu filter: maximum window filtering . . . . .	241
9.4. Image re-sampling . . . . .	242
9.4.1. Interpolation with polynomials. . . . .	243
9.4.2. B-spline interpolation . . . . .	244
9.4.3. Adaptive interpolation. . . . .	246
9.5. Bibliography . . . . .	246
<b>Chapter 10. Image Edge Detection.</b> . . . . .	<b>249</b>
Henri MAÎTRE	
10.1. Continuous edge model . . . . .	250
10.2. Traditional approaches. . . . .	253
10.2.1. Gradient detectors by means of filtering . . . . .	253
10.2.2. Gradient detectors by means of templates . . . . .	253
10.2.3. Pre- and post-processing. . . . .	255
10.2.4. Zero crossing Laplacian detectors . . . . .	256
10.2.5. Detectors by means of template matching . . . . .	258
10.3. Analytical approaches . . . . .	259
10.3.1. Canny's criteria . . . . .	259
10.3.2. Deriche and Shen-Castan filters . . . . .	261
10.3.3. 2D extension. . . . .	262
10.3.4. Variants of analytical filters. . . . .	263
10.4. Active contours (snakes). . . . .	264
10.4.1. Physical approach to edges . . . . .	264
10.4.2. Evolution equation of active contours. . . . .	267
10.4.3. Level sets. . . . .	270
10.4.4. Active geodesic models . . . . .	270
10.5. Edge tracking and closing. . . . .	271
10.5.1. Graph search methods . . . . .	271
10.5.2. Automations . . . . .	274
10.6. Bibliography. . . . .	275
<b>Chapter 11. Region Segmentation</b> . . . . .	<b>279</b>
Henri MAÎTRE	
11.1. Histogram methods. . . . .	279
11.1.1. Using knowledge . . . . .	280

11.1.1.1. Bayesian thresholding . . . . .	280
11.1.1.2. Neyman-Pearson thresholding . . . . .	281
11.1.2. Thresholding without knowledge . . . . .	281
11.1.3. Classification methods . . . . .	282
11.1.4. Selection on histogram and in image . . . . .	283
11.1.5. Selection on histogram and regularization . . . . .	283
11.2. Region transformation methods . . . . .	285
11.2.1. Region growing . . . . .	287
11.2.2. Region splitting . . . . .	288
11.2.3. Region merging . . . . .	289
11.2.4. Pyramids . . . . .	290
11.3. Adjacency graphs . . . . .	291
11.4. MDL (Minimum Description Length) method . . . . .	292
11.5. Mumford and Shah approach . . . . .	294
11.5.1. Formal approach . . . . .	294
11.5.2. Simplifying the Mumford-Shah method . . . . .	296
11.6. Bibliography . . . . .	299
<b>Chapter 12. Textures . . . . .</b>	<b>303</b>
Henri MAÎTRE	
12.1. What is a texture? . . . . .	303
12.1.1. Random or regular distribution? . . . . .	305
12.1.2. Texture scales . . . . .	305
12.1.3. Analysis or synthesis . . . . .	306
12.2. Texture models . . . . .	306
12.2.1. A biologically plausible model . . . . .	306
12.2.2. Stochastic models . . . . .	307
12.3. Texture analysis and recognition . . . . .	307
12.3.1. General outline . . . . .	307
12.3.1.1. Analysis window . . . . .	307
12.3.1.2. Classification of data . . . . .	308
12.3.1.3. Dimension reduction . . . . .	308
12.3.2. Statistical measurement approaches . . . . .	308
12.3.2.1. First-order statistics . . . . .	309
12.3.2.2. Specific point statistics . . . . .	310
12.3.2.3. High-order statistics . . . . .	310
12.3.2.4. Particular pixel configurations . . . . .	313
12.3.3. Energy detection in frequency channels . . . . .	313
12.3.3.1. Low filters . . . . .	314
12.3.3.2. Ring and corner filters . . . . .	314
12.3.3.3. Gabor dyadic filters . . . . .	315
12.3.3.4. Wavelets and QMF . . . . .	315



12.3.4. Optimized filters . . . . .	316
12.3.5. Self-regressive modeling . . . . .	316
12.4. Markov fields approaches . . . . .	317
12.4.1. Manjunath and Chellappa method . . . . .	318
12.4.2. Kervrann and Heitz method . . . . .	319
12.5. Structural methods . . . . .	321
12.6. Heterogenous structures . . . . .	321
12.7. Bibliography . . . . .	322
<b>Chapter 13. Description of Edges and Shapes . . . . .</b>	<b>325</b>
Henri MAÎTRE	
13.1. Characteristic function . . . . .	326
13.2. Forms descriptions . . . . .	327
13.2.1. Representations by moments . . . . .	328
13.2.2. Representations by invariant moments . . . . .	328
13.2.3. Incorporating boxes and minimal boxes . . . . .	329
13.3. Guzman polygons . . . . .	330
13.4. Freeman chains . . . . .	331
13.4.1. Definition . . . . .	332
13.4.2. Properties of Freeman chains . . . . .	333
13.4.3. Shape recognition using Freeman chains . . . . .	336
13.5. Fourier descriptors . . . . .	336
13.5.1. Tangent descriptor . . . . .	336
13.5.2. Complex representation . . . . .	337
13.6. Polynomial approximations . . . . .	339
13.6.1. Approximation of a point cloud using a single straight line . . . . .	340
13.6.1.1. Approximation by linear regression . . . . .	340
13.6.1.2. Approximation by main inertia axis . . . . .	341
13.6.1.3. Robust estimations . . . . .	341
13.6.1.4. Estimation of a combination of straight lines . . . . .	343
13.6.1.5. Estimation extension to cone shapes . . . . .	343
13.6.2. Polygon approximations, simplification of polygon edges . . . . .	343
13.6.2.1. Chord (or Ramer) algorithm . . . . .	344
13.6.2.2. Dunham algorithm . . . . .	344
13.6.2.3. Wahl and Danielsson algorithm . . . . .	344
13.6.2.4. Progressive algorithm . . . . .	345
13.6.3. Approximation using splines . . . . .	345
13.6.3.1. Approximation . . . . .	346
13.6.3.2. Interpolation . . . . .	347
13.7. Hough transform . . . . .	347
13.7.1. Definitions . . . . .	348
13.7.1.1. Transform from l to m . . . . .	348

13.7.1.2. Transform from $m$ to 1 . . . . .	349
13.7.1.3. Crossing from one HT to another . . . . .	350
13.7.1.4. Generalization . . . . .	350
13.7.1.5. HT and adapted filtering . . . . .	350
13.7.2. Implementation of the HT . . . . .	351
13.7.2.1. Discretization of $\mathcal{H}$ . . . . .	351
13.7.2.2. Parameterization of shapes . . . . .	352
13.7.3. Detections using the HT . . . . .	354
13.8. Conclusion . . . . .	354
13.9. Bibliography . . . . .	354
<b>List of Authors</b> . . . . .	<b>357</b>
<b>Index</b> . . . . .	<b>359</b>