## Preface

This book, which deals with vibration in continuous media, originated from the material of lectures given to engineering students of the National Institute of Applied Sciences in Lyon and to students preparing for their Master's degree in acoustics.

The book is addressed to students of mechanical and acoustic formations (engineering students or academics), PhD students and engineers wanting to specialize in the area of dynamic vibrations and, more specifically, towards medium and high frequency problems that are of interest in structural acoustics. Thus, the modal expansion technique used for solving medium frequency problems and the wave decomposition approach that provide solutions at high frequency are presented.

The aim of this work is to facilitate the comprehension of the physical phenomena and prediction methods; moreover, it offers a synthesis of the reference results on the vibrations of beams and plates. We are going to develop three aspects: the derivation of simplified models like beams and plates, the description of the phenomena and the calculation methods for solving vibration problems. An important aim of the book is to help the reader understand the limits hidden behind every simplified model. In order to do that, we propose simple examples comparing different simplified models of the same physical problem (for example, in the study of the transverse vibrations of beams).

The first few chapters are devoted to the general presentation of continuous media vibration and energy method for building simplified models. The vibrations of continuous three-dimensional media are presented in Chapter 1 and the equations which describe their behavior are established thanks to the conservation laws which govern the mechanical media. Chapter 2 presents the problem in terms of variational formulation. This approach is fundamental in order to obtain, in a systematic way, the equations of the simplified models (also called condensed media), such as beams, plates or shells. These simplified continuous media are often sufficient

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models to describe the vibrational behavior of the objects encountered in practice. However, their importance is also linked to the richness of the information which is accessible thanks to the analytical solutions of the equations which characterize them. Nevertheless, since these models are obtained through *a priori* restriction of possible three-dimensional movements and stresses, it is necessary to master the underlying hypothesis well, in order to use them advisedly. The aim of Chapters 3 and 4 is to provide these hypotheses in the case of beams and plates. The derivation of equations is done thanks to the variational formulations based on Reissner and Hamilton's functionals. The latter is the one which is traditionally used, but we have largely employed the former, as the limits of the simplified models obtained in this way are established more easily. This approach is given comprehensive coverage in this book, unlike others books on vibrations, which dedicate very little space to the establishment of simplified models of elastic solids.

Chapters 5, 6 and 7 deal with the different aspects of the behavior of beams and plates in free vibrations. The vibrations modes and the modal decomposition of the response to initial conditions are described, together with the wave approach and the definition of image source linked to the reflections on the limits. We must also insist on the influence of the "secondary effects", such as shearing, in the problems of bending plates. From a general point of view, the discussion of the phenomena is done on two levels: that of the mechanic in terms of modes and that of the acoustician in terms of wave's propagation. The notions of phase speed and group velocity will also be exposed.

We will provide the main analytical results of the vibrations modes of the beams and rectangular or circular plates. For the rectangular plates, even quite simple boundary conditions often do not allow analytical calculations. In this case, we will describe the edge effect method which gives a good approximation for high order modes.

Chapter 8 is dedicated to the introduction of damping. We are going to show that the localized source of damping results in the notion of complex modes and in a difficulty of resolution which is much greater than the one encountered in the case of distributed damping, where the traditional notion of vibrations modes still remains.

The calculation of the forced vibratory response is at the center of two chapters. We will start by discussing the modal decomposition of the response (Chapter 9), where we are going to introduce the classical notions of generalized mass, stiffness and force. Then we will continue with the decomposition in forced waves (Chapter 10) which offers an alternative to the previous method and is very effective for the resolution of beam problems.

For the modal decomposition, the response calculations are conducted in the frequency domain and time domain. The same instances are treated in a manner which aims to highlight the specificities of these two calculation techniques. Finally we will study the convergence of modal series and the way to accelerate it.

In the case of forced wave decomposition, we will show how to treat the case of distributed and non-harmonic excitations, starting from the solution for a localized, harmonic excitation. This will lead us to the notion of integral equation and its key idea: using the solution of a simple case to treat a complicated one.

Chapters 11 and 12 deal with the problem of approximating the solutions of vibration problems, using the Rayleigh-Ritz method. This method employs directly the variational equations of the problems. The classical approach, based on Hamilton's functional, is used and the convergence of the solutions studied is illustrated through some examples. The Rayleigh-Ritz quotient – which stems directly from this approach – is also introduced.

A second approach is proposed, based on the Reissner's functional. This is a method which has not been at the center of accounts in books on vibrations; however, it presents certain advantages, which will be discussed in some examples.